

1. Cinemàtica

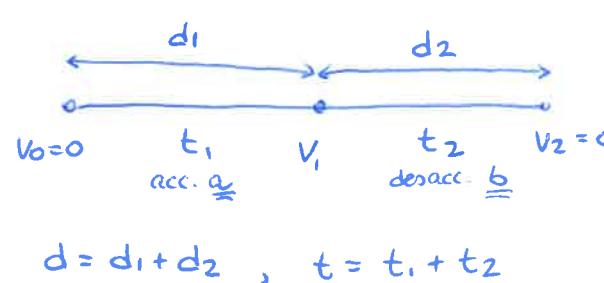
(PROBLEMES)

$$(1) t_{\min} = \left(\frac{2d(a+b)}{ab} \right)^{1/2}$$

d = dist. recomanada

a = acc. màx

b = desacceleració màx



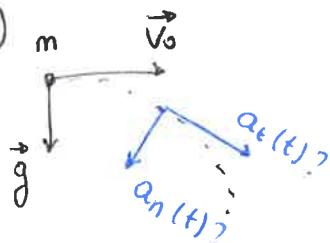
(HRUA)

$$d = v_0 t + \frac{1}{2} a t^2 \rightarrow d_1 + d_2 = \frac{1}{2} a t_1^2 + v_i t_2 - \frac{1}{2} b t_2^2 \Rightarrow t_1 = \sqrt{\frac{2db}{ab+a^2}}$$

$$v = v_0 + at \Rightarrow v_i = at_1 \\ v_2 = 0 = at_1 - bt_2 \quad \left\{ \Rightarrow t_2 = \frac{a}{b} t_1 \right.$$

$$t = t_1 + t_2 = t_1 + \frac{a}{b} t_1 = \dots = \sqrt{\frac{2d(a+b)}{ab}}$$

$$(4) \quad m \quad \vec{v}_0 \quad \vec{a} = (0, -g), \quad \vec{v} = (v_0, -gt)$$



$$v = \sqrt{v_0^2 + g^2 t^2} \rightarrow \frac{dv}{dt} = \frac{g^2 t}{\sqrt{v_0^2 + g^2 t^2}} = a_t(t)$$

$$\text{(Alternativa: } a_t = \vec{a} \cdot \frac{\vec{v}}{v} \text{)}$$

$$\vec{a}_n = \vec{a} - \vec{a}_t = g \frac{\vec{v}_0}{\sqrt{v_0^2 + g^2 t^2}}$$

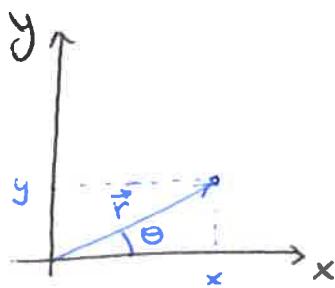
$$\oplus a_n = \sqrt{a^2 - a_t^2} = \sqrt{g^2 - (a_t)^2} =$$

$f(t=1) ?$

$$f = \frac{v^2}{a_n} = \frac{\sqrt{v_0^2 + g^2 t^2}}{g v_0}^3$$

$$\Rightarrow f(1) = \frac{\sqrt{(v_0^2 + g^2)^3}}{g v_0}$$

⑤ Components de \vec{v} i \vec{a} en coord. polars



$$\vec{r}(t) = (r \cos \theta, r \sin \theta)$$

$$\begin{cases} \dot{r} = \frac{dr}{dt} \\ \dot{\theta} = \frac{d\theta}{dt} \end{cases}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = (\dot{r} \cos \theta - r \dot{\theta} \sin \theta, \dot{r} \sin \theta + r \dot{\theta} \cos \theta)$$

$$\vec{v}(t) = \dot{r} (\cos \theta, \sin \theta) + r \dot{\theta} (-\sin \theta, \cos \theta)$$

$$\text{sigui } \hat{r} = (\cos \theta, \sin \theta) \quad i \quad \hat{\theta} = (-\sin \theta, \cos \theta)$$

$$\text{Llavors, } \vec{v}(t) = \dot{r} \hat{r} + r \dot{\theta} \hat{\theta} = \vec{v}_r + v_{\theta},$$

$$\begin{cases} \vec{v}_r = \dot{r} \hat{r} \\ \vec{v}_{\theta} = r \dot{\theta} \hat{\theta} \end{cases}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \ddot{r} \hat{r} + \dot{r} \dot{\theta} \hat{\theta} + r \ddot{\theta} \hat{\theta} + r [\ddot{\theta} \hat{\theta} - \dot{\theta}^2 \hat{r}] =$$

$$\vec{a}(t) = (\ddot{r} - \dot{\theta}^2 r) \hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta}) \hat{\theta}$$

$$\begin{cases} \vec{a}_r = (\ddot{r} - r \dot{\theta}^2) \hat{r} \\ \vec{a}_{\theta} = (2\dot{r}\dot{\theta} + r\ddot{\theta}) \hat{\theta} \end{cases}$$

⑥

MR

$$a = \frac{k}{b-s}$$

$$k, b = \text{ct}$$

$s = \text{dist. recorreguda}$

$s(v) ?$

$a(v) ?$

$$\text{sabem } v = \frac{ds}{dt}, \quad a_t = \frac{dv}{dt}$$

$$\text{MR} \Rightarrow \text{Només hi ha } a_t \Rightarrow a = a_t = \frac{dv}{dt}$$

$$\frac{k}{b-s} = v \frac{dv}{ds} \rightarrow \int_0^s \frac{k}{b-s} ds = \int_0^v v dv \rightarrow$$

$$\rightarrow \frac{1}{2} v^2 = \left[-k \ln(b-s) \right]_0^s = -k \ln \left(\frac{b}{b-s} \right) \rightarrow$$

$$\rightarrow \boxed{s(v) = b \left(1 - e^{-\frac{v^2}{2k}} \right)}$$

$$a(v) = \frac{k}{b - b \left(1 - e^{-\frac{v^2}{2k}} \right)} = \dots \Rightarrow$$

(7)

MC

$$R = 2 \text{ m}$$

$$\Theta(t) = 3t^2 - 2t$$

$$\Theta(4)? \quad S(4)? \quad v(4)? \quad w(4)? \quad a_n(4)? \quad a_t(4)? \quad \alpha(4)?$$

$$\circ \quad \Theta(4) = 40 \text{ rad}$$

$$\circ \quad S(4) = R\Theta = 80 \text{ m}$$

$$\circ \quad v = \frac{ds}{dt} = R(\theta' - 2) \Rightarrow v(4) = 44 \text{ m/s}$$

$$\circ \quad \omega = \frac{d\theta}{dt} = 6t - 2 \Rightarrow \omega(4) = 22 \text{ rad/s}$$

$$\circ \quad a_n = \frac{v^2}{R} = \frac{44^2}{2} = 968 \text{ m/s}^2$$

$$\circ \quad a_t = \frac{dv}{dt} = 6R \Rightarrow a_t(4) = 12 \text{ m/s}^2$$

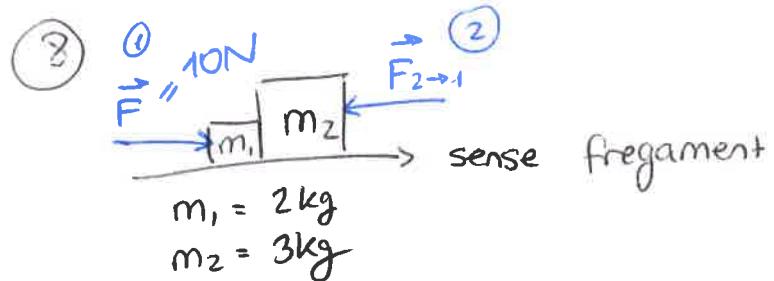
$$\circ \quad \alpha = \frac{d\omega}{dt} = 6 \text{ rad/s}^2$$

obs: S no és la distància recomanada; ja que inicialment el móbil es mou en direcció contrària i a $t = \frac{1}{3}$ canvia el signe de ω .

$$\text{Llavors, dist. rec.} = \int_0^t \|\vec{v}(t)\| dt = \int_0^t R|\omega| dt = \int_0^{1/3} R\omega dt - \int_{1/3}^4 R\omega dt$$

2. Dinàmica

(PROBLEMES)



Doneu la força de contacte entre els blocs.

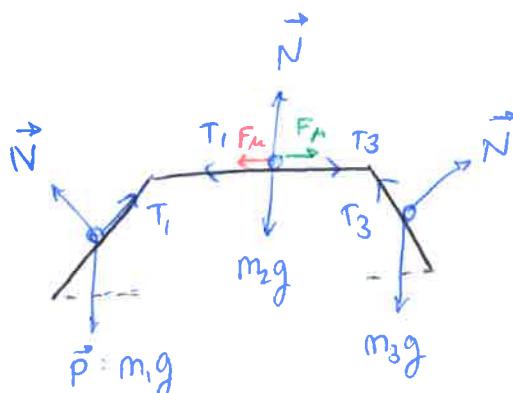
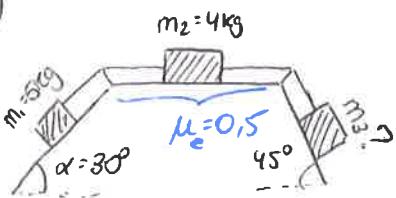
$$\begin{aligned} \vec{F}_{2 \rightarrow 1} &= m_1 \vec{F} \Rightarrow F - F_C = m_1 a \\ \vec{F}_{1 \rightarrow 2} &= m_2 \vec{F} \Rightarrow F_C = m_2 a \end{aligned} \quad \left\{ \Rightarrow a = \frac{F}{m_1 + m_2} \right.$$

① $F_C = \frac{m_2}{m_1 + m_2} F = 6\text{ N}$



② $F_C = \frac{m_1}{m_1 + m_2} F = 4\text{ N}$

⑨



① Suposem $T_3 > T_1$:

$$\sum \vec{F} = 0 \Rightarrow \begin{cases} m_1: T_1 - m_1 g \sin 30^\circ = 0 \rightarrow T_1 = m_1 g \sin 30^\circ \\ m_3: T_3 - m_3 g \sin 45^\circ = 0 \rightarrow T_3 = m_3 g \sin 45^\circ \\ m_2: T_1 + F_\mu - T_3 = 0 \end{cases}$$

$$F_\mu \leq \mu_e N_2 = \mu_e m_2 g$$

$$T_1 + F_\mu - T_3 \geq 0 \rightarrow \mu_e m_2 g \geq m_3 g \sin 45^\circ - m_1 g \sin 30^\circ \rightarrow$$

$$\rightarrow m_3 \leq \frac{\mu_e m_2 + m_1 \sin 30^\circ}{\sin 45^\circ} = \frac{9/2}{\sqrt{2}/2} = \frac{9}{2}\sqrt{2} \text{ kg}$$

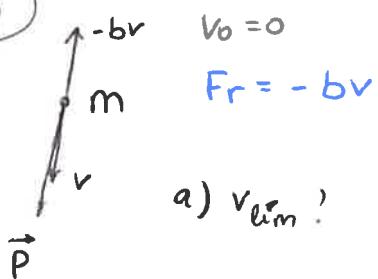


② Suposem $T_1 > T_3$

$$T_1 \leq T_3 + F_{\mu_{\max}} \rightarrow m_1 g \sin 30^\circ \leq m_3 g \sin 45^\circ + \mu m_2 g \rightarrow \\ \rightarrow m_3 \geq \frac{m_1 \sin 30^\circ - \mu m_2}{\sin 45^\circ} = \frac{\frac{1}{2}}{\sqrt{2}/2} = \frac{\sqrt{2}}{2} \text{ kg}$$

Llavors $\frac{1}{2}\sqrt{2} \leq m_3 \leq \frac{9}{2}\sqrt{2}$.

(10)



a) v_{\lim} ? $\sum F = ma \rightarrow ma = mg - bv \rightarrow$

$$\rightarrow a = g - \frac{b}{m}v = g - \gamma v$$

Volem v_{\lim} , és a dir, a $a = 0$.

$$\boxed{v_{\lim} = \frac{g}{\gamma} = \frac{m}{b}g}$$

b) $v(t)$? $\Delta y(t)$?

$$\left. \begin{array}{l} a = g - \gamma v \\ a = \frac{dv}{dt} \end{array} \right\} \Rightarrow \frac{dv}{dt} = g - \gamma v \rightarrow \int_0^v \frac{dv}{g - \gamma t} = \int_0^t dt \rightarrow$$

$$\rightarrow \left[-\frac{1}{\gamma} \ln(g - \gamma v) \right]_0^v = [t]_0^t \rightarrow \ln\left(\frac{g - \gamma v}{g}\right) = -\gamma t \Rightarrow$$

$$\Rightarrow \boxed{v(t) = v_{\lim} (1 - e^{-\gamma t})}$$

$$\Delta y(t) = \int_0^t v dt = \int_0^t v_{\lim} (1 - e^{-\gamma t}) dt = v_{\lim} t - v_{\lim} \int_0^t e^{-\gamma t} dt = v_{\lim} t - v_{\lim} \left(\frac{e^{-\gamma t}}{-\gamma} - \frac{1}{-\gamma} \right) \\ = v_{\lim} t - \frac{v_{\lim}}{\gamma} (1 - e^{-\gamma t})$$

c) t? tq $v(t) = \frac{V_{\text{lim}}}{2}$. $\Delta y(t)$?

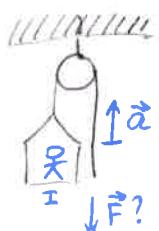
$$v(t) = V_{\text{lim}} \left(1 - e^{-\gamma t}\right) = \frac{V_{\text{lim}}}{2} \rightarrow e^{-\gamma t} = \frac{1}{2} \rightarrow t = \frac{1}{\gamma} \ln 2$$

$$\Delta y\left(\frac{1}{\gamma} \ln 2\right) = V_m \left(\frac{\ln 2}{\gamma}\right) - \frac{V_m}{\gamma} \left(1 - e^{-\gamma \frac{\ln 2}{\gamma}}\right) = \frac{V_m}{\gamma} \left(\ln 2 - \frac{1}{2}\right)$$

d) $\Delta y(t) = H$

$$V_m t - \frac{V_m}{\gamma} \left(1 - e^{-\gamma t}\right) = H$$

11



Sigui $\vec{F}_{A \rightarrow B}$ la força que fa A sobre B.

$$\vec{F}_{T \rightarrow I} + \vec{F}_{c \rightarrow I} + \vec{F}_{c' \rightarrow I} = M\vec{a} \quad (\text{sobre l'individu})$$

$$\vec{F}_{T \rightarrow c} + \vec{F}_{c' \rightarrow c} + \vec{F}_{I \rightarrow c} = m\vec{a} \quad (\text{sobre la cabina})$$

Per la 3a llei de Newton,

$$\vec{F}_{I \rightarrow c} = -\vec{F}_{c \rightarrow I}, \quad |\vec{F}_{I \rightarrow c}| = N$$

$$\vec{F}_{c' \rightarrow I} = -\vec{F}_{I \rightarrow c'},$$

Suposem que la tensió de la corda és la mateixa a tot arreu.

$$\text{Llavors, } |\vec{F}_{I \rightarrow c}| = |\vec{F}_{c' \rightarrow c}| = T$$

Sobre l' individu:

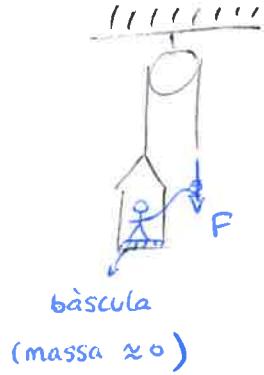
$$-Mg + N + T = M\vec{a}$$

Sobre la cabina:

$$-mg - N + T = m\vec{a}$$

$$\begin{cases} -Mg + N + T = M\vec{a} \\ -mg - N + T = m\vec{a} \end{cases} \rightarrow T = \frac{1}{2}(M+m)(\vec{a} + g)$$

Suposem ara que hi ha una bàscula:



sobre la bàscula:

$$\vec{F}_{T \rightarrow B} + \vec{F}_{I \rightarrow B} + \vec{F}_{c \rightarrow B} = m' a \approx 0$$

ss
O

$m' \approx 0$

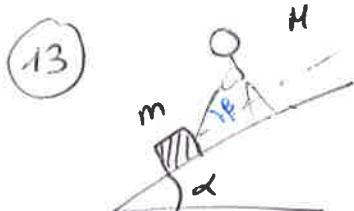
$$|\vec{F}_{I \rightarrow B}| = |\vec{F}_{c \rightarrow B}| = |\vec{F}_{B \rightarrow c}| = N'$$

" "

$$|\vec{F}_{B \rightarrow I}|$$

Les egs. escalars corresponents són:

$$\begin{aligned} -Mg + N' + T &= Ma \\ -mg - N' + T &= ma \end{aligned} \quad \left. \begin{array}{l} T \text{ trobada (és el mateix sistema que abans)} \\ |N| = \frac{1}{2}(M-m)(a+g) \end{array} \right\}$$



$$|\vec{F}_{N \rightarrow c}| = 100N = |\vec{F}_{c \rightarrow T}| = T$$

Puja a $v = ct$.

$$Mg = 500N$$

$$mg = 200N$$

$$\alpha = 15^\circ$$

$$\beta = 30^\circ$$

2a Llei Newton:

$$\vec{F}_{T \rightarrow T'} + \vec{F}_{P \rightarrow T'} + \vec{F}_{c \rightarrow T'} = 0$$

$v. ct.$

$$\vec{F}_{P \rightarrow T'} = N + \vec{F}_f, \quad |\vec{F}_f| \leq \mu N$$

En les direccions paral·leles al pla:

$$-mgs \sin \alpha - \mu_d N + T \cos \beta = 0$$

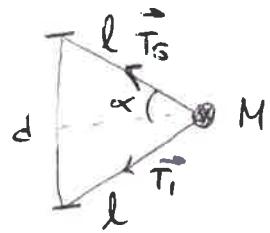
En les perpendiculars:

$$-mg \cos \alpha + N + T \sin \beta = 0$$

$$\Rightarrow \mu_d = 0,24$$

$$|\vec{F}_{P \rightarrow N}| = \sqrt{N^2 + (\mu N')^2} = \dots = 532,9N \quad \mu_e \doteq \mu = 0,405$$

(15)



$$\vec{T}_s + \vec{T}_i + \vec{F}_{T \rightarrow M} = M \vec{a}$$

Ho escrivim en components:

$$l = 1.5 \text{ m}$$

$$\underline{\text{Horitz:}} \quad T \cos \alpha + T_i \cos \alpha = MR\omega^2$$

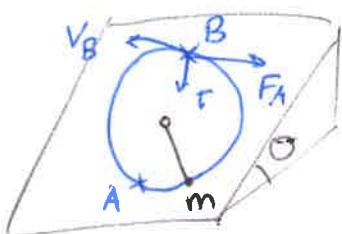
$$d = 2.5 \text{ m}$$

$$T = 180 \text{ N}$$

$$\underline{\text{Vert:}} \quad T \sin \alpha - T_i \sin \alpha - Mg = 0$$

2. Dinàmica d'un sistema de partícules. Treball i Energia (PROBLEMES)

(4)



$$\theta = 30^\circ, m = 2 \text{ kg}, L = 1 \text{ m}, \mu = 0.1$$

$$v_A = 10 \text{ m/s}, v_B? T_B?$$

$$E_{\text{mec}_A} = \frac{1}{2} m v_A^2 \quad (\text{Prenent A l'origen del potencial})$$

$$E_{\text{mec}_B} = \frac{1}{2} m v_B^2 + mgh$$

$$E_A - E_B = W_{\text{FNC}} = W_\mu = \int_A^B \vec{F}_\mu = -\mu mg \cos \theta \pi L$$

Llavors, aillant v_B :

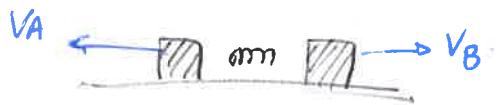
$$v_B = \sqrt{v_A^2 - 4gL \sin \theta - 2\mu g \pi L \cos \theta} = 8.66 \text{ m/s}$$

Per obtenir T_B , apliquem $\sum \vec{F} = ma$ en el punt B

$$a_n = \frac{v^2}{R}$$

(7)

(7)



$$m_A = 1 \text{ kg} \quad v_B = 0,5 \text{ m/s}$$

$$m_B = 2 \text{ kg} \quad E_{\text{Pmolla}} ?$$

$$\mathcal{U} = \frac{1}{2} k l^2, \quad k = \text{ct recuperadora}$$

$l = \text{long, recuperade}$

$$\mathcal{U} = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \quad \rightarrow \quad \vec{F}_{\text{ext}} = \vec{F}_A + \vec{F}_B = 0 \quad \left. \right\} (\vec{P} \text{ ct})$$

$$\vec{F}_{\text{ext}} = \frac{d\vec{P}}{dt}$$

Com que es conserva la quantitat de moviment:

$$\vec{P} = m_A \vec{v}_A + m_B \vec{v}_B$$

$$0 = m_A v_A + m_B v_B \quad \Rightarrow \quad v_A = -\frac{m_B}{m_A} v_B$$

Llavors, $\boxed{\mathcal{U} = \frac{1}{2} m_A \left(-\frac{m_B}{m_A} v_B \right)^2 + \frac{1}{2} m_B v_B^2 =}$

$$= \frac{1}{2} \left(1 + \frac{m_B}{m_A} \right) m_B v_B^2 = \frac{1}{2} (1+2) 2 (0,5)^2 = 0,75 \text{ J}$$

8

$$m_1 = 1 \text{ kg}$$

$$m_2 = 2 \text{ kg}$$

$$m_3 = 3 \text{ kg}$$

$$\vec{F}_{\text{ext}}^{\text{ext}} \quad t \quad \vec{P} = 2t^3 \hat{i} + 4t \hat{j}$$

$$\vec{r}_{\text{cm}}(0) = \vec{i} + 2\vec{j}$$

$$E_{\text{C}}(3) = 500 \text{ J}$$

$$\vec{V}_{\text{cm}}?$$

$$\vec{r}_{\text{cm}}(t)?$$

$$\vec{F}?$$

$$\vec{a}_{\text{cm}}(t)?$$

$$E_{\text{Cinterna}}(3)?$$

$\vec{F}_{\text{conserv.}} \rightarrow$ deriva d'una U externa

$$E_{\text{mec}} = K_{\text{total}}^{\text{Ec}} + U_{\text{ext}}^{\text{Ep ext}}$$

(Pot no conservar-se si \exists forces internes no conservatives)

Si les forces internes són conservatives:

$$E = K_{\text{total}} + U_{\text{ext}} + U_{\text{int}}$$

en conserva.

Recordem:

$$a) \vec{r}_{\text{cm}} = \frac{\sum_{i=1}^3 m_i \vec{r}_i}{\sum_{i=1}^3 m_i}$$

$$b) \vec{P} = M \vec{V}_{\text{cm}}$$

$$\vec{P} = M \vec{V}_{\text{cm}} \rightarrow \vec{V}_{\text{cm}} = \frac{\vec{P}}{M} = \frac{1}{3} t^3 \hat{i} + \frac{2}{3} t \hat{j}$$

$$\vec{r}_{\text{cm}}(t) - \vec{r}_{\text{cm}}(0) = \int \vec{V}_{\text{cm}} dt = \frac{1}{12} t^4 \hat{i} + \frac{1}{3} t^2 \hat{j} + \vec{r}_{\text{cm}}(0) = \left(1 + \frac{1}{12} t^4, 2 + \frac{1}{3} t^2 \right)$$

$$\vec{F}_{\text{ext}} = \frac{d\vec{P}}{dt} = M \vec{a}_{\text{cm}}$$

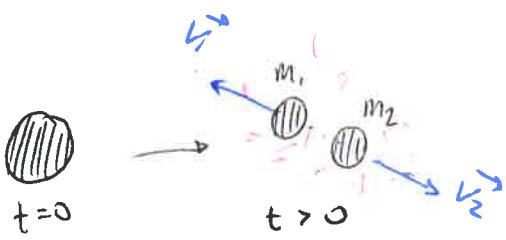
$$\frac{d\vec{P}(t)}{dt} = \vec{F}_{\text{ext}} = (6t^2, 4) \rightarrow \vec{a}_{\text{cm}} = \frac{\vec{F}_{\text{ext}}}{M} = (t^2, 2/3)$$

$$K_{\text{total}} = \sum_{i=1}^3 \frac{1}{2} m_i v_i^2 = \frac{1}{2} M \vec{V}_{\text{cm}}^2 + \sum_{i=1}^3 \frac{1}{2} m_i v_{i,\text{cm}}^2,$$

$$\text{on } v_{i,\text{cm}} = \vec{v}_i - \vec{V}_{\text{cm}}$$

III
 K_{int}

(9)



$$k_i = E_C \text{ de } m_i \quad (i=1,2) \quad = \frac{k_i}{k_{\text{total}}} = \frac{\frac{1}{2} m_i v_i^2}{\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2}$$

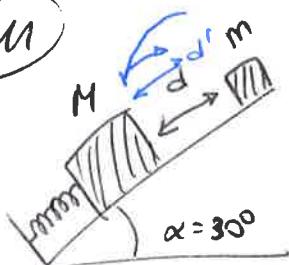
Com que no actuen forces externes:

$$\vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2 \text{ es conserva} \implies \vec{P}_{\text{ini}} = 0 = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

Llavors, $m_1 v_1 = m_2 v_2 \implies v_2 = \frac{m_1}{m_2} v_1$

$$\frac{k_i}{k_T} = \frac{\frac{1}{2} m_i v_i^2}{\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2} v_1\right)^2} = \frac{m_i}{m_1 + \frac{m_i^2}{m_2}} = \boxed{\frac{m_2}{m_2 + m_1} = \frac{k_i}{k_{\text{total}}}}$$

(11)



$$k_{\text{molla}} = 104 \text{ N/m}$$

$$d = 4 \text{ m}, \quad d' = 2.5 \text{ m}$$

$$m = 1 \text{ kg} \quad f_{\text{max}} = 3 \text{ cm} \quad (\text{compressió addicional})$$

$$\mu = 0$$

① càcul vel. m abans de xocar. amb M. (v_i)

$$E_{\text{ini}} = k_{\text{ini}} + U_{\text{gravitat}}^{\text{ini}} \quad \left. \begin{array}{l} \\ \\ mg 4 \sin 30^\circ = 2mg \end{array} \right\}$$

$$2mg = \frac{1}{2} m v_i^2 \rightarrow$$

$$\rightarrow v_i = 2\sqrt{g}$$

$$E_{\text{final}} = k_{\text{fin}} + U_{\text{gravitat}}^{\text{fin}} \quad \left. \begin{array}{l} \\ \\ \frac{1}{2} m v_i^2 \end{array} \right\}$$



② Càlcul de la V de m després del xoc (v_f)

$$\left. \begin{array}{l} E_{\text{ini}} = \frac{1}{2} m v_f^2 \\ E_{\text{fin}} = mg(2,5) \sin 30^\circ \end{array} \right\} \quad \boxed{v_f = \sqrt{2,5g}}$$

③ Càlcul vel. de M després del xoc (V)

Suposem ΔP de les forces externes és negligible

Llavors, P_{total} es conserva en el xoc.

$$\vec{F}_{\text{ext}} = \frac{d\vec{P}}{dt} \rightarrow \Delta \vec{P} = \int_0^T \vec{F}_{\text{ext}} \cdot dt \approx 0$$

$$\begin{aligned} \text{Llavors, } P_{\text{ini}} &= P_{\text{fin}} \\ " & " \\ mv_i & -mv_f + MV \end{aligned} \Rightarrow$$

$$\Rightarrow MV = mv_i + mv_f \Rightarrow V = \frac{m(v_i + v_f)}{M}$$

④ Aplico principi conserv. ($E_{\text{mecc}_M} + E_{\text{el. molla}}$) després del xoc.

t_{ini} després del xoc: i final a max. compressió de la molla.

δ_{max} : max. compressió addicional de la molla.

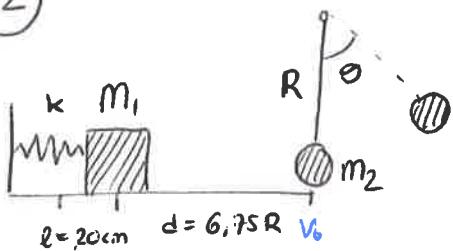
δ_0 : compressió inicial molla

$$\begin{aligned} t_0: \rightarrow E_{\text{ini}} &= \frac{1}{2} MV^2 + 0 + \frac{1}{2} k \delta_0^2 \\ t_f \rightarrow E_f &= 0 = mg \delta_{\text{max}} \sin \alpha + \frac{1}{2} k (\delta_{\text{max}} + \delta_0)^2 \end{aligned} \quad (\underline{\alpha = 30^\circ})$$

$$\text{Llavors, } E_{\text{ini}} = E_f \Rightarrow \frac{1}{2} MV^2 = \frac{1}{2} k \delta_{\text{max}}^2 \Rightarrow$$

$$\Rightarrow M \left(\frac{m(v_i + v_f)}{M} \right)^2 = k \delta_{\text{max}}^2 \rightarrow M = \frac{m^2 (v_i + v_f)^2}{k \delta_{\text{max}}^2}$$

(12)



$$m_1 = 100 \text{ g}$$

$$R = 80 \text{ cm}$$

$$\theta = 90^\circ$$



$$v_2 = \frac{-2v_0}{3} \leftarrow \boxed{\text{Ball}} \rightarrow v_2 = \frac{1}{3}v_0$$

$\text{a) } m_2?$ xoc elàstic? $v_0 = 3\sqrt{2gR}$?

Com que es conserva la quantitat de moviment:

$$m_1 v_0 = m_1 \left(-\frac{2}{3} v_0 \right) + m_2 \left(\frac{v_0}{3} \right) \rightarrow \boxed{m_2 = 5m_1}$$

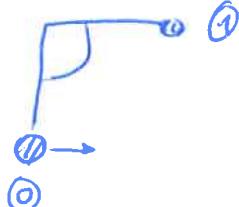
xoc elàstic? ($E_c = ct$)?

$$-\frac{2}{3}v_0 - \frac{v_0}{3} = -v_0$$

coef. percussió: $e = -\frac{v_{\text{rel}, f}}{v_{\text{rel}, i}} = -\frac{v_0}{v_0} = 1 \Rightarrow \boxed{\text{xoc elàstic}}$

$\therefore v_0 - 0 = v_0$

$v_0?$



$$E_{c0} = E_{p1} \rightarrow \frac{1}{2} m_2 v_2^2 = m_2 g R \rightarrow$$

$$\rightarrow v_2 = \sqrt{2gR} \rightarrow \boxed{v_0 = 3\sqrt{2gR}}$$

b) $\mu = 0,5$

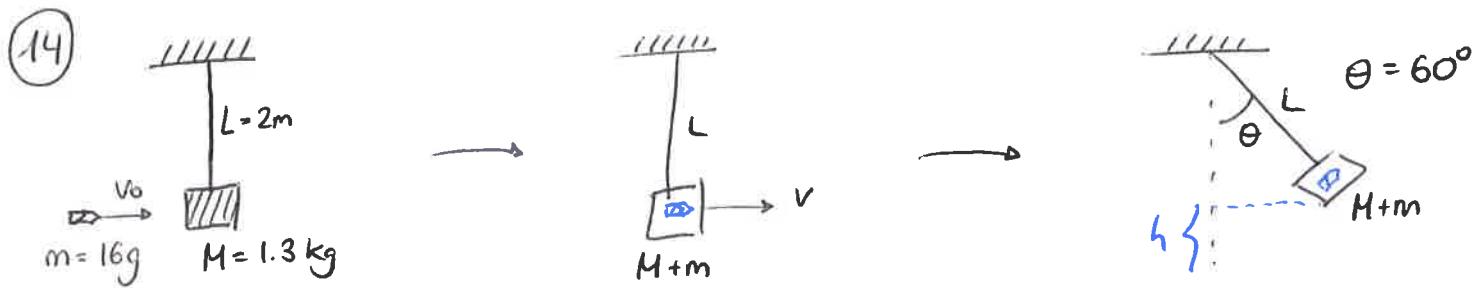
$w_\mu?$

$$\boxed{w_\mu = -F_\mu \Delta x = -\mu m_1 g \cdot 7R = -2,74 \text{ J}}$$

$k?$

$$E_{p1} = E_{cf} + w_\mu \rightarrow \frac{1}{2} k l^2 = \frac{1}{2} m_1 v_0^2 + w_\mu \rightarrow$$

$$\rightarrow \boxed{k = \frac{m_1 v_0^2 + 2w_\mu}{l^2} = 490 \text{ N/m}}$$



a) v_0 ?

Per conservació del moment lineal després de l'impacte (xoc inelàstic):

$$mv_0 = (M+m)v \rightarrow v_0 = \frac{M+m}{m} v$$

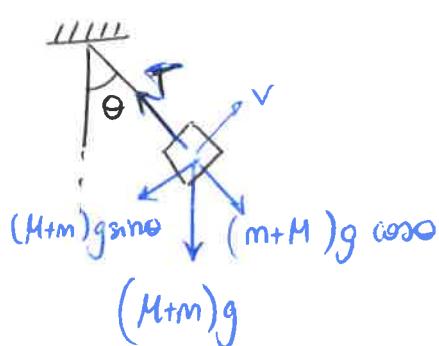
$$E_{C_0} = E_{P_h} \rightarrow \frac{1}{2}(M+m)v^2 = (M+m)gh \rightarrow$$

$$v = \sqrt{2gh} = \sqrt{2g(L-L\cos\theta)} = \sqrt{2gL(1-\cos\theta)} = \sqrt{gL}$$

$$v_0 = \frac{M+m}{m} \sqrt{gL} = 364,14 \text{ m/s}$$

b) α ? ω ? si $\theta = 30^\circ$

T ?



$$\begin{aligned} (\sum F_x = m \cdot a_x) \\ (M+m)g \sin\theta &= (M+m)a_t = (M+m)L\alpha \rightarrow \\ \alpha &= \frac{g}{L} \sin\theta = 2,45 \text{ rad/s}^2 \end{aligned}$$

$$T - (M+m)g \cos\theta = (M+m)a_n = (M+m)L\omega^2 \quad \frac{v^2}{L}$$

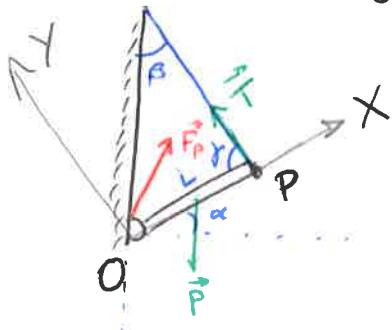
Trobarem v quan $\theta = 30^\circ$:

$$\begin{aligned} E_{C_0} &= E_{C_0} + E_{P_0} \rightarrow \frac{1}{2}(M+m)v^2 = \frac{1}{2}(M+m)v_0^2 + (M+m)gh_\theta \rightarrow \\ \rightarrow v_0 &= \sqrt{v^2 - 2gh_\theta} = \sqrt{gL(2\cos 30^\circ - 1)} = 3,79 \text{ m/s} \end{aligned}$$

Llavors, $T = 20,67 \text{ N}$

16

$$m = 20 \text{ kg}, L = 2 \text{ m}, \alpha = 30^\circ, \beta = 45^\circ$$



$\sum F = 0$ NO! \vec{P} el centre de masses es mou

Llavors, les cond. d'equilibri per un sólid rígid:

i) $\vec{F}_{\text{net}} = 0$ si CM té $v=0$ o $v=ct$

ii) $M_{\text{net}} = 0$

i) $\vec{F}^{\text{net}} = \vec{T} + \vec{mg} + \vec{F}_p = (-T\cos\gamma, T\sin\gamma, 0) + m(-g\sin\alpha, -g\cos\alpha, 0) + (F_x, F_y, 0)$

a) $-T\cos\gamma - mg\sin\alpha + F_x = 0$

b) $T\sin\gamma - mg\cos\alpha + F_y = 0$

ii) calculem el moment de forces net:

[Nota: \vec{M}_O de \vec{F}]

Si Q és pt. d'aplicació de \vec{F} ,

$$\vec{M}_O = \overrightarrow{OQ} \times \vec{F}$$

$$\begin{aligned} \vec{M}_O^{\text{net}} &= \overrightarrow{OP} \times \vec{T} + \overrightarrow{O(\text{CM})} \times \vec{mg} + \cancel{\overrightarrow{OQ} \times \vec{F}_p} = \\ &= (0, 0, L T \sin\gamma) + (0, 0, -\frac{L}{2} mg \cos\alpha) \end{aligned}$$

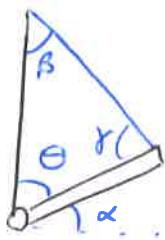
Llavors,

c) $L T \sin\gamma - \frac{L}{2} mg \cos\alpha = 0$

$$\left. \begin{array}{l} a) \\ b) \\ c) \end{array} \right\} \dots \quad \begin{array}{l} T = \\ \rightarrow F_x = 62 \text{ N} \\ \rightarrow F_y = 133 \text{ N} \end{array}$$



b) α_{30° ? ω_{90° ?



Equacions dinàmiques d'un sòlid rígid:

- $M\vec{a}_{CM} = \vec{F}_{\text{neta}} \quad \left(= \frac{d\vec{P}}{dt} \right) \rightarrow \text{descriu mov. translatori del CM}$
- $\frac{d\vec{\omega}}{dt} = \vec{M}_O^{\text{net}} \quad (\text{si } O \text{ fix o CM})$

⊗ En el cas particular d'aquest exercici, el moviment és pla i, en aquest cas, (ii) queda:

$$\frac{d\vec{\omega}}{dt} = \vec{M}_O^{\text{net}} \quad (\text{els dos tenen direcció perpendicular al mov.})$$

Si el sòlid descriu rotació d'un eix fix o d'un eix que passa per CM però de direcció fixa:

$$L_O = I_O \omega = I_O \dot{\theta}$$

↪ moment inèrcia resp. eix (panta per O)

Sabem I_O d'una barra respecte un extrem és:

$$I = \frac{1}{3} m L^2$$

Per tant, tenim $L_O = \frac{1}{3} m L^2 \dot{\theta}$

$$\text{A més, } M_O^{\text{net}} = \frac{L}{2} mg \cos \alpha = \frac{L}{2} mg \sin \theta$$

↪ degut al pes.

Per ii), tenim:

$$\frac{1}{3} m L^2 \dot{\theta} = \frac{L}{2} mg \sin \theta \rightarrow \ddot{\theta}(0) = \frac{3}{2} \frac{g}{L} \sin \theta(0) = \frac{3}{2} \frac{g}{L} \sin 60^\circ$$

$$\boxed{\frac{3\sqrt{3}}{4} \frac{g}{L}}$$

Per trobar $\dot{\theta}_{90^\circ}$, utilitzem la conservació de l'energia $E_{mec, \text{barra}}$
 (Es conserva pq \vec{F}_p no fa treball pq és punt fix)

$$E_m^i = E_p^i + E_c^i = E_{p,CM}^i = Mgh_{CM}^i = Mg \frac{L}{2} \sin 30^\circ = \frac{1}{4} MgL$$

$$E_m^f = E_p^f + E_c^f = \frac{1}{2} I_0 w_f^2 + Mg y_{CM}^f = \frac{1}{2} \left(\frac{1}{3} ML^2 \right) w_f^2 + \left(-\frac{1}{2} MgL \right) =$$

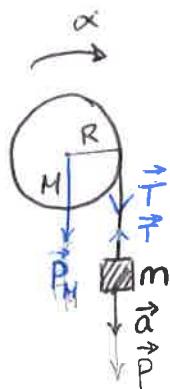
$$= \frac{1}{6} ML^2 w_f^2 - \frac{1}{2} MgL$$

$$E_m^i = E_m^f \Rightarrow \frac{1}{4} MgL = \frac{1}{6} ML^2 w_f^2 - \frac{1}{2} MgL$$

$$\rightarrow [w =]$$

I_0 barra
resp eix rotació
" "
 $\frac{1}{3} ML^2$

(18) $\alpha?$ $R=0,5m$, $M=25\text{ kg}$, $m=10\text{ kg}$, $\vec{a}?$



$$M_o = I_0 \alpha \rightarrow TR = I_0 \alpha$$

$$I_0 = \int_0^R r^2 dm = \left[dm = \sigma 2\pi r dr \right] = 2\pi \sigma \int_0^R r^3 dr =$$

$$= \frac{1}{2} \pi \sigma R^2 R^2 = \frac{1}{2} MR^2$$

$$\text{Llavors, } M_o = TR = \frac{1}{2} MR^2 \alpha \rightarrow T = \frac{1}{2} MR\alpha$$

$$\text{Prenem el moviment de } m: mg - T = ma$$

$$a = \alpha R$$

=>

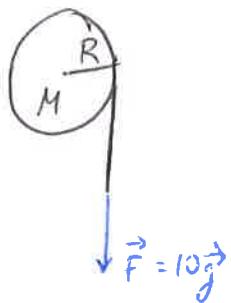
$$\Rightarrow \left[\alpha = \frac{m}{M + \frac{1}{2}M} \frac{g}{R} = 8,7 \text{ rad/s}^2 \right]$$

$$\left[a = \alpha R = 4,36 \text{ m/s}^2 \right]$$

$$\left[T = \frac{Mm}{2M+M} g = 54,45 \text{ N} \right]$$

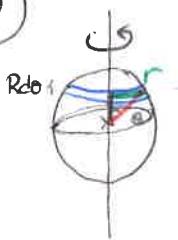
(18) b) En comptes de la massa, fer una força $\vec{F} = \vec{P}_m$.

$$\sum M = I_0 R \rightarrow FR = I_0 R^2 = \frac{1}{2} MR^2 \omega \rightarrow$$



$$\rightarrow \alpha = \frac{2F}{MR} = 15.7 \text{ rad/s}^2$$

(20)



$$I = \frac{2}{3} m R^2$$

$$\rightarrow []_{R \sin \theta}^{2\pi r} R d\theta$$

$$dm = \sigma 2\pi r R d\theta$$

$$R \sin \theta = r$$

$$dm = \sigma 2\pi R^2 \sin \theta d\theta$$

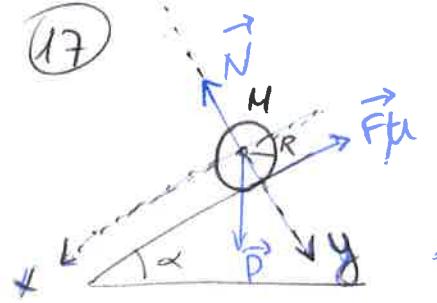
$$I = \int_0^\pi r^2 dm = \int_0^\pi R^2 \sin^2 \theta \sigma 2\pi R^2 \sin \theta d\theta =$$

$$= R^4 \sigma 2\pi \int_0^\pi \sin^3 \theta d\theta = R^4 \sigma 2\pi \int_0^\pi \sin \theta (1 - \cos^2 \theta) d\theta =$$

$$= 2\pi \sigma R^4 \left[-\cos \theta \right]_0^\pi = 2\pi \sigma R^4 \left[-\frac{1}{3} \cos^3 \theta \right]_0^\pi = \sigma \cdot V = m$$

$$= 4\pi \sigma R^4 - \frac{4}{3} \pi \sigma R^4 = \frac{8}{3} \pi \sigma R^4 = \frac{2}{3} (4\pi R^2 \sigma) R^2 = \frac{2}{3} m R^2$$

(17)



$$M = 0.5 \text{ kg}, R = 10 \text{ cm}, \alpha = 30^\circ$$

\vec{a}_{cm} ? α_{cm} ? F_μ ?

$$\star M\vec{a}_{cm} = \vec{F}_{ext}$$

$$\text{En l'eix } x: M\vec{a}_{cm} = \vec{F}_x^{ext} = Mg \sin \alpha - \vec{F}_\mu$$

$$\star M\vec{a}_{cm}^{ext} = I_{cm}\vec{\alpha}_{cm}$$

$$\star \vec{M}_o = \vec{OP} \times \vec{F}$$

$$M_{cm} = F_\mu R = \left(\frac{2}{5}MR^2\right) \cdot \alpha_{cm} \rightarrow \frac{2}{5}MR^2\alpha = RF_\mu$$

I_{cm} : Moment
inèrta esfera resp
eix pel centre $= \frac{2}{5}MR^2$

Com que roda senserelliscar:

La distància recorreguda pel CM quan gira un angle φ és $R\varphi$

$$\rightarrow a_{cm} = R\alpha$$

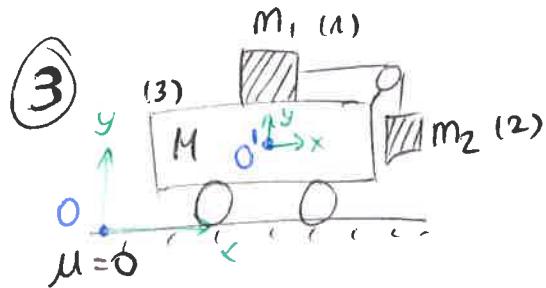
$$\begin{cases} M\vec{a}_{cm} = Mg \sin \alpha - \vec{F}_\mu \\ \frac{2}{5}MR^2\alpha = \vec{F}_\mu \\ a_{cm} = R\alpha \end{cases} \Rightarrow \begin{cases} a_{cm} = \frac{5}{14}g \\ F_\mu = \frac{1}{7}mg \\ \alpha = \end{cases}$$

\vec{v}_{cm} quan ha baixat 2,5 m?

$$x_{cm}(t) = \frac{1}{2}a_{cm}t^2 = 2.5 \rightarrow t = \sqrt{\frac{5}{a_{cm}}}$$

$$v_{cm}(t) = a_{cm} \cdot t \rightarrow v_{cm}(t) = \sqrt{5a_{cm}}$$

3. Canvis de sistema de referència



$\left\{ \begin{array}{l} v, a' \text{ de } m_1 \text{ resp } S' \\ V, A \text{ de } M \text{ resp } S \end{array} \right.$

a) \vec{v}, \vec{a} de cada massa resp S

pq w entre
Si $S' = 0$

$$\vec{r}_i = \vec{OO'} + \vec{r}'_i \quad \rightarrow \quad \boxed{\vec{v}_i = \frac{d\vec{OO'}}{dt} + \frac{d\vec{r}'_i}{dt} = \vec{V} + \vec{v}'_i}$$

$$\rightarrow \vec{a}_i = \vec{A} + \vec{a}'_i$$

$$\vec{v}_3 = (V, 0, 0)$$

$$\vec{v}_1 = \vec{V} + \vec{v}'_1 = (V, 0, 0) + (v', 0, 0)$$

$$\vec{v}_2 = \vec{V} + \vec{v}'_2 = (V, 0, 0) + (0, -v', 0)$$

$$\vec{a}_3 = (A, 0, 0)$$

$$\vec{a}_1 = (A, 0, 0) + (a, 0, 0)$$

$$\vec{a}_2 = (A, 0, 0) + (0, -a, 0)$$

$$b) P_x^{\text{total}} = ct \iff F_x^{\text{ext}} = 0$$

$$\vec{P} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$$

Les forces externes que actuen són el pes del conjunt i la normal que fan les rodes.

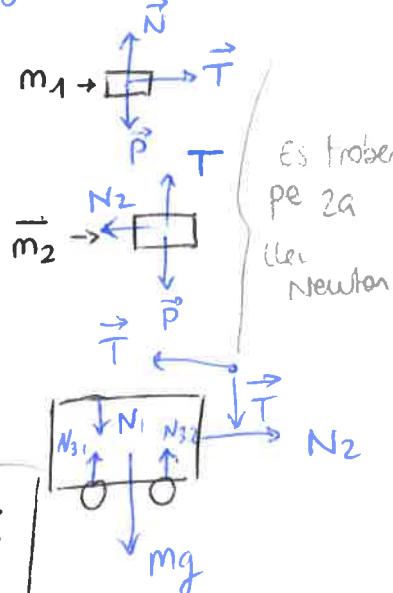
$$\vec{P} - \vec{N} = 0$$

Per trobar $A(\ddot{a})$:

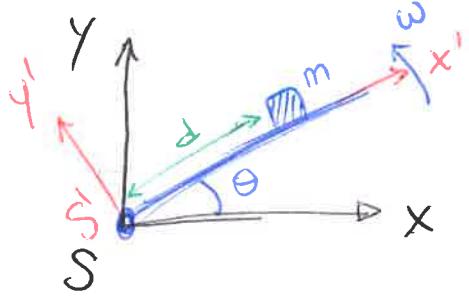
$$\vec{P}_x = \sum m_i \vec{v}_i = m(V + v) + mV + MV = ct$$

$$\Rightarrow \frac{d\vec{P}_x}{dt} = m(a + A) + mA + MA = 0 \Rightarrow \boxed{A = \frac{-m}{2m+M} \ddot{a}}$$

Diagrama de forces:



⑤ $\theta(t) = 2t^2$, $d(t) = 1-t^2$, $S' = \text{rotació del filferro}$



a) $\vec{r}'(t)$, $\vec{v}'(t)$, $\vec{a}'(t)$ de m?

en base S_0

$t \frac{?}{?}$ si $\Theta = 30^\circ$

$$\vec{r}'(t) = (1-t^2, 0, 0)_{S'}$$

$$\vec{v}'(t) = (-2t, 0, 0)_{S'}$$

$$\vec{a}'(t) = (-2, 0, 0)_{S'}$$

obs. que $\begin{cases} \vec{i}' = \cos\theta \vec{i} + \sin\theta \vec{j} \\ \vec{j}' = \sin\theta \vec{i} + \cos\theta \vec{j} \end{cases}$

$$\Rightarrow \begin{cases} \vec{r}'(t) = (1-t^2)(\cos\theta \vec{i} + \sin\theta \vec{j}) \\ \vec{v}'(t) = -2t(\cos\theta \vec{i} + \sin\theta \vec{j}) \\ \vec{a}'(t) = -2(\cos\theta \vec{i} + \sin\theta \vec{j}) \end{cases}$$

Volem $\Theta = 30^\circ$.

Llavors, $\theta(t) = 2t^2 = \frac{\pi}{6} \Rightarrow t = \sqrt{\frac{\pi}{12}}$

$$\begin{cases} \vec{r}_{30^\circ} = \vec{r}'\left(\sqrt{\frac{\pi}{12}}\right) = \left(1 - \frac{\pi}{12}\right)\left(\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right) \\ \vec{v}_{30^\circ} = \vec{v}'\left(\sqrt{\frac{\pi}{12}}\right) = -\sqrt{\frac{\pi}{3}}\left(\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right) \\ \vec{a}_{30^\circ} = \vec{a}'\left(\sqrt{\frac{\pi}{12}}\right) = -\sqrt{3}\vec{i} - \vec{j} \end{cases}$$



↳

b) $m = 1 \text{ kg}$: \vec{F}_{cor} ? \vec{F}_{eul} ? $\vec{F}_{\text{cif.}}$? en base S'

$$\vec{F}_{\text{cor}} = -2m \vec{\omega} \times \vec{v} = -2m \dot{\theta} \vec{k} \times (-2t) \underbrace{(\cos \theta \vec{i} + \sin \theta \vec{j})}_{\vec{i}'} \\ \Rightarrow \boxed{\vec{F}_{\text{cor}} = -2(4t \vec{k}') \times (-2t \vec{i}') = \underline{16t^2 \vec{j}'}}$$

$$\vec{F}_{\text{euler}} = -m \vec{\alpha} \times \vec{r}' = -4 \vec{k}' \times (1-t^2) \vec{i}' = -4(1-t^2) \vec{j}'$$

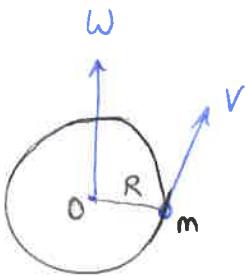
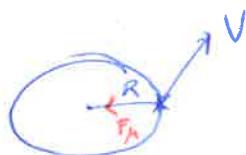
$$\vec{F}_{\text{cif}} = -m \vec{\omega} \times (\vec{\omega} \times \vec{r}') = \underline{16t^2(1-t^2) \vec{i}'}$$

c) $\vec{F}_{\text{total real}}$?

$$\text{Sabem } m\vec{a}' = \vec{F}_{\text{real}} + \vec{F}_{\text{eul}} + \vec{F}_{\text{cor}} + \vec{F}_{\text{cif.}}$$

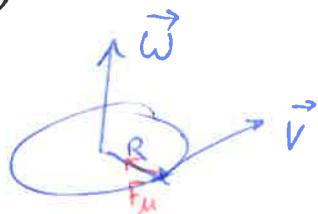
$$\Rightarrow \boxed{\vec{F}_{\text{real}} = \vec{m}\vec{a}' - (\vec{F}_{\text{eul}} + \vec{F}_{\text{cor}} + \vec{F}_{\text{cif.}}) = \\ = -(2+16t^2(1-t^2)) \vec{i}' + 4(1-5t^2) \vec{j}'}$$

8

a) $\omega = 0$. $a?$ $F_\mu?$ 

$$a = a_n = \frac{v^2}{R}$$

$$F_\mu = ma_n = m \frac{v^2}{R}$$

b) $\vec{\omega} = ct.$ $\vec{F}_{\text{inercia}}?$ $\vec{F}_\mu?$ 

$$\vec{F}_{\text{Eul}} = -m\vec{\alpha} \times \vec{r}' = 0 \quad (\alpha = 0)$$

$$\vec{F}_{\text{cor}} = -2m\vec{\omega} \times \vec{v}' = -2m\omega V$$

$$\vec{\omega} \perp \vec{v}'$$

$$\vec{F}_{\text{uf}} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r}) = -m\omega^2 R$$

$$\vec{\omega} \perp \vec{r}$$

$$m\vec{a}' = \vec{F}_{\text{real}} + \vec{F}_{\text{cor}} + \vec{F}_{\text{eul}} + \vec{F}_{\text{uf}} =$$

$$m \frac{v^2}{R} = F_\mu - 2m\omega V + 0 - m\omega^2 R \Rightarrow$$

$$\Rightarrow F_\mu = m \frac{(V + \omega R)^2}{R}$$

c) $\vec{\omega} = -ct,$

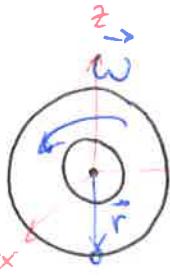
Analogamente,

$$F_{\text{cor}} = 2m\omega V$$

$$F_{\text{uf}} = -m\omega^2 R$$

$$F_\mu = \frac{m(V - \omega R)^2}{R}$$

9



a) $\vec{\omega} = (\omega, 0, 0)$, $\vec{r} = (0, 0, -R)$

$$m\vec{a}' = \vec{F}_{\text{real}} - \vec{F}_{\text{cif}} - \vec{F}_{\text{cor}} - \vec{F}_{\text{Eul}} - \vec{F}_{\text{translatio}}$$

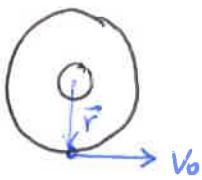
$$\vec{a}_{S'} = \frac{\vec{F}}{m} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2\vec{\omega} \times \vec{v}'$$

$$\vec{a}_{S'} = \frac{\vec{F}}{m} - \omega^2 R \cdot \vec{k} \rightarrow 0 = \frac{\vec{N}}{m} - \omega^2 R$$

$$\vec{N} = \omega^2 R m = mg'$$

$$g' = \omega^2 R \rightarrow \frac{g}{2} = \omega^2 R \rightarrow \boxed{\omega = \sqrt{\frac{g}{2R}}}$$

b)



$$\vec{a}_{S'} \Rightarrow \frac{v_0^2}{R} = \frac{\vec{N}'}{m} - \underbrace{\omega^2 R \vec{k}}_{\vec{F}_{\text{cif}}} - \underbrace{2\omega v_0 \vec{k}}_{\vec{F}_{\text{cor}}} \rightarrow$$

$$\rightarrow N' = m \left(\frac{v_0^2}{R} + \omega^2 R + 2\omega v_0 \right) = m \left(\frac{v_0}{R} + \omega \right)^2 R$$

Comparat amb la normal anterior: $N = m\omega^2 R$

$N' > N \Rightarrow$ Nota una força gravitòria més gran.

Si haguéssim suposat que gira en sentit oposat a la rotació ω , llavors:

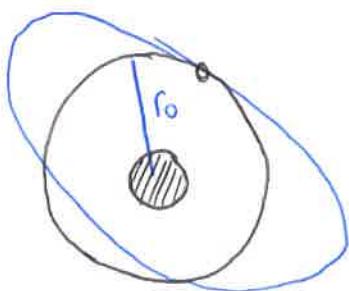
$$N'' = m \left(\frac{v_0}{R} - \omega \right)^2 R \Rightarrow N'' < N$$

(Força gravitòria més lleugera)

4 Camp gravitatori

(PROBLEMES)

3



$$L_{0\text{ ini}} \xrightarrow{\text{encerc molar}} L_{0\text{ fin}} = \frac{L_{0\text{ ini}}}{2}$$

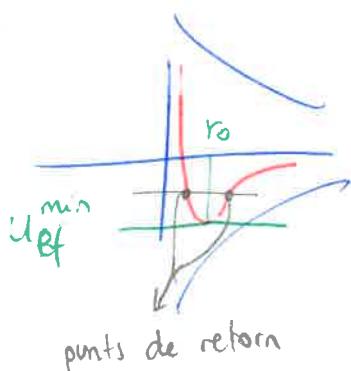
$$\Delta E = ct$$

$$\text{Sabem } E_{\text{mec}} = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r}$$

U_{ef}

* Si $U_{\text{ef}}^{\min} = E_{\text{mec}}$ \rightarrow traj. circular

* Si $U_{\text{ef}}^{\min} < E_{\text{mec}} < 0$ \rightarrow traj. elíptica



Busquem els punts de retorn de l'òrbita
(Corresponen als punts de $E_{\text{mec}} = U_{\text{ef}}$)

$$E_{\text{mec}}^{\text{ini}} = \frac{L_{\text{ini}}^2}{2mr_0^2} - \frac{GMm}{r_0}$$

$$E_{\text{mec}}^{\text{fin}} = \frac{1}{2} m \dot{r}^2 + \frac{L_{\text{fin}}^2}{2mr^2} - \frac{GMm}{r}$$

$$\text{Volem } E_{\text{mec}}^{\text{ini}} = E_{\text{mec}}^{\text{fin}} \Rightarrow \frac{L_{\text{ini}}^2}{2mr_0^2} - \frac{GMm}{r_0} = \frac{1}{2} m \dot{r}^2 + \frac{L_{\text{fin}}^2}{2mr^2} - \frac{GMm}{r}$$

Els punts de retorn són a $\dot{r}=0$:

$$\frac{L_i^2}{2mr_0^2} - \frac{GMm}{r_0} = \frac{L_f^2}{2mr^2} - \frac{GMm}{r} \quad (*)$$

Calculem L_i en funció de r_0 : $\frac{dU_{\text{ef}}}{dr} = 0$ en $r = r_0$

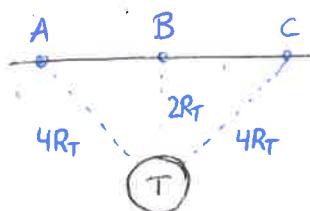
$$U_{\text{ef}}^{\text{ini}}(r) = \frac{L_i^2}{2mr^2} - \frac{GMm}{r} \rightarrow \frac{dU_{\text{ef}}^{\text{ini}}}{dr} = \frac{-L_i^2}{mr^3} + \frac{GMm}{r^2} = 0$$

$$\hookrightarrow \text{Llavors, obtenim } L_{\text{ini}}^2 = GMm^2 r_0$$

Substituint a ④ i resolent, obtenim:

$$r = r_0 \left(i \pm \frac{\sqrt{3}}{2} \right)$$

⑥



a) Com que segueix una trajectòria rectilínia a $v = ct$:

$$\vec{F}_{\text{neta}} = 0 \rightarrow \vec{F}_{\text{grav}} = -\vec{F}_{\text{motor}}$$

$$\vec{F}_g(A) = -G \frac{Mm}{(4R_T)^2} \vec{u}_A \implies \vec{F}_{\text{mot}}(A) = G \frac{Mm}{(4R_T)^2}$$

$$U_g(P) = - \int_0^P \vec{F}_g d\vec{r}$$

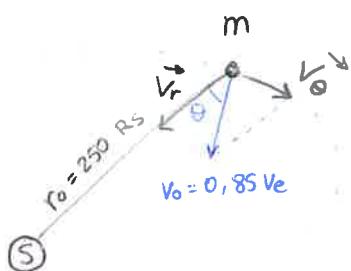
$$\begin{aligned} b) W_{A \rightarrow B}^{\text{motor}} &= \int_A^B \vec{F}_{\text{motor}}(\vec{r}) d\vec{r} = - \int_A^B \vec{F}_g(\vec{r}) d\vec{r} = - \left(\int_A^0 + \int_0^B \right) = \\ &= \int_0^A \vec{F}_{\text{grav}} d\vec{r} - \int_0^B \vec{F}_{\text{grav}} d\vec{r} = -U_g(A) + U_g(B) = \\ &= - \left(-G \frac{Mm}{4R_T} \right) + \left(-G \frac{Mm}{2R_T} \right) = \end{aligned}$$

$$W_{B \rightarrow C}^{\text{motor}} = -W_{A \rightarrow B}^{\text{mot}} \quad (\text{per simetria})$$

$$c) \text{Cond. de trajectòria el·líptica} \rightarrow E_{\text{mec}} = \frac{1}{2}mv^2 - G \frac{Mm}{r} < 0$$

Llavors, r ha de ser el mínim possible per que quedi més negatiu, però en el cas que v sigui molt gran, pot no ser possible.

(7)

a) v_r ? v_θ ? k ? U ? L_o ?

$$\vec{v} = \dot{r} \hat{r} + r \dot{\theta} \hat{\theta}$$

$$v_r = v_0 \cos \theta = 0,85 v_e \cos \theta$$

$$v_\theta = v_0 \sin \theta = 0,85 v_e \sin \theta$$

$$v_e ? \implies \frac{1}{2} m v_e^2 = \frac{GMm}{r_0} \rightarrow v_e = \sqrt{\frac{GM}{125 R_s}}$$

Alejándose, $\dot{r} = 0,85 \sqrt{\frac{GM}{125 R_s}} \cos 53^\circ$

$$\dot{\theta} = 0,85 \sqrt{\frac{GM}{125 R_s}} \sin 53^\circ$$

$$K = \frac{1}{2} m (0,85^2 \cdot \frac{GM}{125 R_s})$$

$$U = -\frac{GMm}{250 R_s}$$

$$\vec{L}_o = \vec{r} \times m \vec{v}$$

$$L_o = r_0 m v_e 0,85 \sin 53^\circ$$

$$L_o = 250 R_s m 0,85 \sqrt{\frac{GM}{125 R_s}} \sin 53^\circ \text{ (constant)}$$

$$E = K + U < 0 \Rightarrow \underline{\text{órbita tancada}}$$

$$(constant) \quad \sqrt{-1.11 \cdot 10^{-3} \frac{GMm}{R_s}}$$

b) r_{\min} ?

$$E = \frac{1}{2} m \dot{r}^2 + U_{\text{ef}}$$

$$U_{\text{ef}} = \frac{L^2}{2mr^2} - \frac{GMm}{r}$$

$$\text{Si } r = r_{\min}, \dot{r} = 0$$

$$\text{Llavors, } E = \frac{L^2}{2mr_{\min}^2} - \frac{GMm}{r_{\min}}$$

$$\xrightarrow{\text{alíjelo } r} r_{\min} = \frac{R_s \pm 0,699 R_s}{2,22 \cdot 10^3} \Rightarrow$$

$$r_{\min} = \begin{cases} 765,3 R_s \\ 135,6 R_s \end{cases} \rightarrow \begin{cases} R_{\max} = 765 R_s \\ R_{\min} \approx 136 R_s \end{cases}$$

c) V_{\max} ?

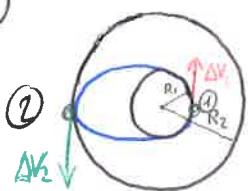
Observem que $V = V_{\max}$ quan $r = r_{\min}$

$$\left\{ \begin{array}{l} E = \frac{1}{2}mv^2 - \frac{GMm}{r} \text{ constant.} \\ L = mr\dot{\theta} \text{ constant.} \end{array} \right.$$

$$L = m r_{\min} \dot{\theta}_{\max} \rightarrow \dot{\theta}_{\max} = V_{\max} = 0,112 \sqrt{\frac{GM}{R_s}}$$

$r_{\min} \rightarrow \dot{r} = 0$

⑧



Δv_1 ? Δv_2 ?

② órbita circular. $m a_n = F_g \rightarrow m v^2/R = -\frac{GMm}{R^2} \Rightarrow$

$$\rightarrow v = \sqrt{\frac{GM}{R}} \Rightarrow \left\{ \begin{array}{l} v_1 = \sqrt{\frac{GM}{R_1}} \\ v_2 = \sqrt{\frac{GM}{R_2}} \end{array} \right.$$

órbita elíptica:

$$\left. \begin{array}{l} ① \rightarrow v_1 + \Delta v_1 \\ ② \rightarrow v_2 - \Delta v_2 \end{array} \right\} \text{Es pot sumar directament ja que tenen la mateixa direcció i sentit.}$$

llavors, per conservació de L :

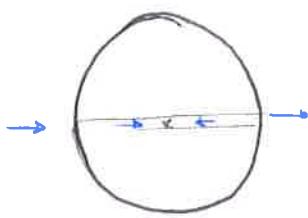
$$L = mR_1(v_1 + \Delta v_1) = mR_2(v_2 - \Delta v_2)$$

Per conservació d' E : $E = \frac{1}{2}m(v_1 + \Delta v_1)^2 - \frac{GMm}{R_1} = \frac{1}{2}m(v_2 - \Delta v_2)^2 - \frac{GMm}{R_2}$

Resolent el sistema:

$$\begin{aligned} v_2 - \Delta v_2 &= \frac{R_1}{R_2} (v_1 + \Delta v_1) \Rightarrow \\ \Rightarrow \frac{1}{2}m(v_1 + \Delta v_1)^2 - \frac{GMm}{R_1} &= \frac{1}{2}m\left(\frac{R_1}{R_2}(v_1 + \Delta v_1)\right)^2 - \frac{GMm}{R_2} \Rightarrow \\ \Rightarrow & \left\{ \begin{array}{l} \Delta v_1 = \sqrt{\frac{GM}{R_1}} \left(\sqrt{\frac{2R_2}{R_1 + R_2}} - 1 \right) \\ \Delta v_2 = \sqrt{\frac{GM}{R_2}} \left(1 - \sqrt{\frac{2R_1}{R_1 + R_2}} \right) \end{array} \right. \end{aligned}$$

11



$$m\ddot{x} = - \frac{GMm}{R_T^3} x \rightarrow \ddot{x} = -\omega^2 x$$

$$\Rightarrow \omega = \sqrt{\frac{GM}{R_T^3}} \rightarrow T = 2\pi \sqrt{\frac{R_T^3}{GM_T}} \approx \underline{\underline{85 \text{ min}}}$$

$$x = A \cos \omega t$$

$$\dot{x} = -R_T \omega \sin \omega t$$

$$\ddot{x} = -R_T \omega^2 \cos \omega t$$

$$\vec{g}(r) = - \frac{GM_T}{R_T^3} \vec{r}$$

$$\vec{g}(R_T) = - \frac{GM}{R_T^2} \vec{u}_r$$

$$V_0 = 7,9 \text{ km/s}$$

$$W = \int_0^{R_T} \frac{GMm}{R_T^3} r dr = \frac{1}{2} \frac{GMm}{R_T}$$

Busquem el potencial a tot punt de l'espai:

$$U(r) = - \int - \frac{GM}{R^3} r dr = \frac{1}{2} G \frac{M_T}{R^3} r^2 + C \quad (r \in [0, R_T])$$

$$U(r) = - \int - G \frac{M}{r^2} dr = - \frac{GM}{r} \quad (r > R_T)$$

Aleshores,

$$U(r) = \begin{cases} \frac{GM_T}{R_T} \left(\frac{r^2}{2R_T^2} - \frac{3}{2} \right) & r \leq R_T \\ - \frac{GM_T}{r} & r > R_T \end{cases}$$

$$W = U(0) - U(R_T) = -\frac{3}{2} \frac{GM}{R_T} + \frac{GM_T}{R_T} = -\frac{1}{2} \frac{GM}{r^2}$$

(9)

 r_{\min}, r_{\max} obreidea) $v_{r_{\min}}, v_{r_{\max}}, \omega_{r \rightarrow s}$?

$$L = m r_{\min} V_{\max} = m r_{\max} v_{\min} \rightarrow v_{\min} = \frac{r_{\min}}{r_{\max}} v_{\max}$$

$$E = \frac{1}{2} m v_{\max}^2 + \frac{2GMm}{r_{\min}} = \frac{1}{2} m v_{\min}^2 + \frac{2GMm}{r_{\max}^2} \rightarrow$$

$$\rightarrow v_{\min}^2 = v_{\max}^2 + 2GM \left(\frac{1}{r_{\max}} - \frac{1}{r_{\min}} \right) \rightarrow$$

$$\rightarrow v_{\min}^2 = \frac{r_{\max}^2}{r_{\min}^2} v_{\max}^2 + 2GM \left(\frac{1}{r_{\max}} - \frac{1}{r_{\min}} \right) \Rightarrow$$

$$\Rightarrow v_{\min}^2 \left(1 - \frac{r_{\max}^2}{r_{\min}^2} \right) = 2GM \left(\frac{1}{r_{\max}} - \frac{1}{r_{\min}} \right) \rightarrow$$

$$\rightarrow v_{\min}^2 = 2GM \left(\frac{1}{r_{\max}} - \frac{1}{r_{\min}} \right) \cdot \frac{1}{1 - \frac{r_{\max}^2}{r_{\min}^2}} =$$

$$= 2GM \cdot \frac{\frac{r_{\min} - r_{\max}}{r_{\min} r_{\max}}}{\frac{r_{\min}^2 - r_{\max}^2}{r_{\min}^2}} = 2GM \cdot \frac{\frac{r_{\min}^2 (r_{\min} - r_{\max})}{r_{\min} r_{\max}}}{r_{\min} r_{\max} (r_{\min}^2 - r_{\max}^2)} =$$

$$= \frac{2GM r_{\min}}{r_{\max} (r_{\min} + r_{\max})} \rightarrow \boxed{v_{\min} = \sqrt{\frac{2GM r_{\min}}{r_{\max} (r_{\min} + r_{\max})}}}$$

$$\boxed{\omega_p = \frac{V}{R} \rightarrow \frac{V_{\max}}{r_{\min}}}$$

$$\boxed{\omega_a = \frac{v_{\min}}{r_{\max}}}$$

b) $\ddot{\theta}(r)$? $v(r)$?

$$\left\{ \begin{array}{l} v = \dot{r} + r\dot{\theta} \\ E = \frac{1}{2}\dot{r}^2m + \frac{L^2}{2mr^2} - \frac{k}{r} = \frac{1}{2}mv^2 + \frac{GMm}{r} \\ L = rm\dot{\theta} \\ r_{\min}v_{\max} = r\dot{m}\dot{\theta} \end{array} \right. \rightarrow \boxed{\dot{\theta} = \frac{r_{\min}}{r} v_{\max}}$$

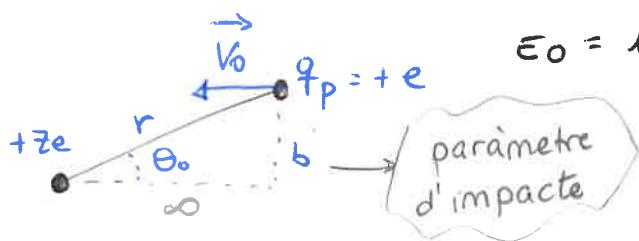
$$v = \dot{r} + r\dot{\theta}$$

$$\frac{1}{2}v_{\max}^2 - \frac{2GM}{r_{\min}} = \frac{1}{2}v^2 - \frac{2GM}{r} \rightarrow v = \sqrt{v_{\max}^2 + 2GM\left(\frac{1}{r} - \frac{1}{r_{\min}}\right)}$$

5. Electroestàtica

(PROBLEMES)

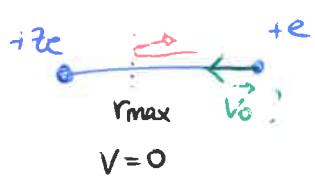
②



$$\epsilon_0 = 10 \text{ MeV}, q_{\text{nuclei}} = +ze, z = 79$$

$K = 9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2}$	$U = qV$
$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J}$	

a) r_{\max} ? $b = 0$



$$F = k \frac{q_1 q_2}{r^2} \quad \text{forsa de Coulomb conservativa.}$$

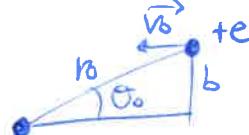
$$\Rightarrow U = k \frac{q_1 q_2}{r}$$

* $E_c + E_p = ct \Rightarrow E_f - E_0 = 0 \Rightarrow k \frac{ze^2}{r_{\max}} = E_0 \Rightarrow$

$$\Rightarrow r_{\max} = \frac{k ze^2}{E_0} = \frac{9e9 \cdot 79 \cdot (1.6 \cdot 10^{-19})^2}{10 \cdot 10^6 \cdot 1.6 \cdot 10^{-19}} = 1.14 \cdot 10^{-14} = 11.4 \text{ fm}$$

fentòmetres = fermi

b) $b = 50 \text{ fm. } r?$



sabem que $E = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} + U(r)$

Volem r_{\max} . $\Rightarrow V_{r_{\max}} = r + \theta$

$$\sin \theta = \frac{b}{r}$$

Sabem L constant. $\Rightarrow \vec{L} = m \vec{r} \times \vec{v} \Rightarrow L = mrV \sin \theta = mv_0 b$

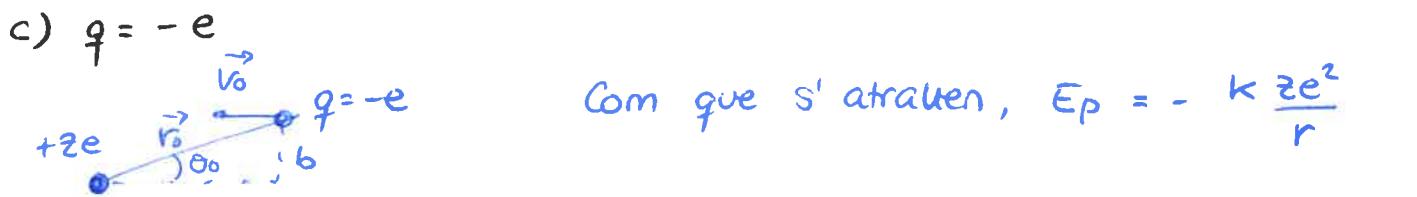
E constant. $\Rightarrow E = \frac{(mv_0 b)^2}{2mr^2} + k \frac{ze^2}{r} = E_0 = \frac{k ze^2}{r_{\max}}$

$$\Rightarrow r^2 - Rr - b^2 = 0 \Rightarrow r = \frac{R \pm \sqrt{R^2 + 4b^2}}{2}$$

$$\Rightarrow r = \frac{R}{2} + \sqrt{\left(\frac{R}{2}\right)^2 + b^2}$$

$$\Rightarrow r = \frac{R}{2} - \sqrt{\left(\frac{R}{2}\right)^2 + b^2}$$

$$\Rightarrow r = 50 \text{ fm}$$

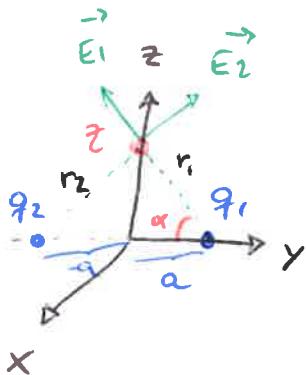


Llavors, $\bar{E} = \frac{L^2}{2mr^2} - \frac{kze^2}{r} = E_0$ ($L = mv_0 b$ per (b))

$$\Rightarrow \frac{(mv_0b)^2}{2mr^2} - \frac{kze^2}{r} = \frac{kze^2}{R} \Rightarrow r^2 + Rr - b^2 = 0 \rightarrow$$

$$\rightarrow r = \begin{cases} -\frac{R}{2} + \sqrt{\left(\frac{R}{2}\right)^2 + b^2} \\ -\frac{R}{2} - \sqrt{\left(\frac{R}{2}\right)^2 + b^2} \end{cases} = 44.6 \text{ fm}$$

③



a) \vec{E} a \vec{Oz} ?

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

sigui $z \in \vec{Oz}$, $\vec{U}_{E_1} = \vec{U}_{q_1, z}$
 $\vec{U}_{E_2} = \vec{U}_{q_2, z}$

$$\vec{E}_1 = k \frac{q_1}{r_1^2} \hat{r}_1, \quad \vec{E}_2 = k \frac{q_2}{r_2^2} \hat{r}_2 \quad i \quad r = r_1 = r_2$$

$$\sin \alpha = \frac{z}{r}$$

Llavors, $E_y = E_{1,y} + E_{2,y} = 0$

$$E_z = E_{1,z} + E_{2,z} = 2E_{1,z} = 2(E_1 \sin \alpha) = 2E_1 \left(\frac{z}{r} \right)$$

$$= 2k \frac{q_1}{r^2} \left(\frac{z}{r} \right) = \frac{2kq_1 z}{r^3} = \frac{2kq_1 z}{(\sqrt{z^2 + r^2})^3}$$

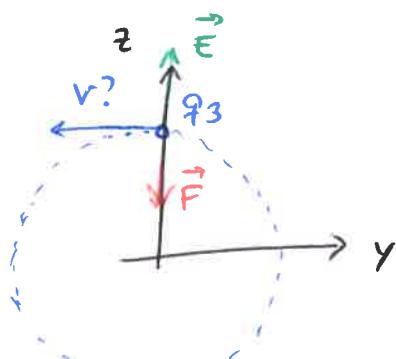
$$r_1^2 = a^2 + z^2$$

b) punts de XZ en que $\vec{E} = E_{\max}$?

$$\frac{dE}{dz} = 0 \Rightarrow \frac{d}{dz} \left(\frac{2kqz}{(\sqrt{z^2+a^2})^3} \right) = 2kq \left(\frac{1}{(\sqrt{z^2+a^2})^3} + z \left(\frac{-3}{2} \right) \left(\frac{2z}{(\sqrt{z^2+a^2})^5} \right) \right) = 2kq \frac{z^2+a^2 - 3z^2}{(\sqrt{z^2+a^2})^5} = 0 \Rightarrow a^2 + 2z^2 = 0 \Rightarrow z = \frac{1}{\sqrt{2}}a$$

c) $q_3 = -q$, massa m , $\vec{r}_0 = (0, 0, R)$

v ? pq descriuen circumferència?



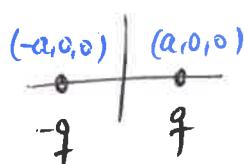
òrbita circular:

$$\begin{aligned} \vec{F} &= m\vec{a}_n, \quad a_n = \frac{v^2}{R} \rightarrow F = m \frac{v^2}{R} \\ \vec{F} &= q_3 \vec{E} \rightarrow F = \frac{2kq^2 R}{(R^2 + a^2)^{3/2}} \quad (z=R) \end{aligned}$$

$$\Rightarrow \frac{2kq^2 R}{(R^2 + a^2)^{3/2}} = m \frac{v^2}{R} \Rightarrow v^2 = \frac{2kq^2 R^2}{m} (R^2 + a^2)^{3/2}$$

⑤

Pot. i camp en \vec{r} creat per q a \vec{a}



$$V_{q, \vec{a}}(\vec{r}) = \frac{kq}{\|\vec{r} - \vec{a}\|}$$

$$\vec{E}_{q, \vec{a}}(\vec{r}) = \frac{kq}{\|\vec{r} - \vec{a}\|^3} (\vec{r} - \vec{a})$$

+ principi superpos.

i) Pot. efectiu a $(x, 0, 0)$

$$V_{\text{total}}(x, 0, 0) = V_{q, \vec{a}}(x, 0, 0) + V_{-q, \vec{a}}(x, 0, 0) = \frac{kq}{|x-a|} - \frac{kq}{|x+a|}$$



←

ii) Força sobre e col·locada a $(l, 0, 0)$

$$\vec{F}_{\text{total}} = e \vec{E}_{\text{total}}(l, 0, 0) = e \left(\vec{E}_{q, \vec{a}}(l, 0, 0) + \vec{E}_{q, \vec{a}'}(l, 0, 0) \right) =$$

$$= \dots = \frac{ekq4al}{(l^2a^2)^2} = \frac{ekq4a}{l^3} \frac{1}{\left(1 - \left(\frac{a^2}{l^2}\right)\right)^2} \underset{l \gg a}{\approx} \frac{ekq4a}{l^3} \vec{i}$$

iii) Signe de e per que s'allunyi:

$$\begin{array}{ll} \text{Si } l > 0 & \text{cal que } e > 0 \\ l < 0 & e < 0 \end{array}$$

iv) Velocitat amb que arriba a l'infinít

si inicialment està a $(l, 0, 0)$ ($l > 0$) en repòs:

Per conservació de l'energia:

$$E_{\text{mec}}^{\text{ini}} = ekq \left(\frac{1}{|l-a|} - \frac{1}{|l+a|} \right) = \frac{ekq2a}{l^2-a^2}$$

$$E_{\text{mec}}^{\text{fin}} = \frac{1}{2} m v_{\infty}^2 \quad l \gg a$$

Igualant:

$$\frac{1}{2} m v_{\infty}^2 = \frac{ekq2a}{l^2-a^2} \rightarrow v_{\infty} = \sqrt{\frac{4ekqa}{m(l^2-a^2)}} \approx \sqrt{\frac{4ekqa}{ml^2}}$$

⑦ La força neta de:

$$\vec{F} = \frac{ekq}{l^2} \left[\frac{1}{(1-\frac{q}{e})^2} + \frac{1}{(1+\frac{q}{e})^2} - \frac{2}{\sqrt{(1-\frac{q^2}{e^2})^3}} \right] \vec{i}$$

q q

$$f(x) = \frac{1}{(1-x)^2} + \frac{1}{(1+x)^2} - \frac{2}{\sqrt{(1+x^2)^3}} \approx qx^2 + \dots$$

↳ Taylor
at $x=0$

NO ACABAT

⑧ Dipol $\vec{P} = (0, p, 0)$ → (Distribució de càrrega a l'origen que crea un potencial)

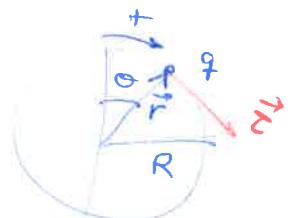
$$V_{\text{dipol}}(\vec{r}) = k \frac{\vec{r} \cdot \vec{P}}{r^3}$$

$$\vec{E}_{\text{dipol}}(\vec{r}) = k \left(\frac{3(\vec{P} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{P}}{r^3} \right)$$

{ El que es crea lluny de la distribució }

(i) Pot. creat per \vec{P} en funció de θ

$$\vec{r} = R (\sin \theta, \cos \theta, 1), \quad \vec{P} = (0, p, 0)$$



$$\Rightarrow V_{\text{dipol}}(\theta) = k \frac{R p \cos \theta}{R^3}$$

$$\text{ii)} \vec{F}_{\text{dipol}} = q \vec{E}_{\text{dipol}} = \frac{k q P}{R^3} (3 \cos \theta \sin \theta, 3 \cos^2 \theta - 1, 0)$$

iii) Signe de q per tal que força atractiva

$$\text{si } \theta = 0, \quad \vec{F}_{\text{dipol}}(\vec{r}) = \frac{k q P}{R^3} (0, 2, 0) \quad \begin{array}{l} \text{Per tant, atractiva} \\ \text{si } q < 0 \end{array} \quad \vec{F}_{\text{dipol}}$$

iv) Posicions equilibrí

$$\text{Corresponen a } \sum \vec{F} = 0. \quad \vec{F}_{\text{dipol}} + \vec{F}_{\text{anella}}$$

si suposem que $\mu = 0$ ($\Rightarrow \vec{F}_{\text{anella}} + \alpha$ l'anell)

$$\begin{cases} (F_{\text{neto}})_t = 0 \\ (F_{\text{neto}})_n = 0 \\ F_{\text{dipol}} = -F_{\text{anella}} \end{cases}$$

6

Busquem posicions on $(\vec{F}_{\text{dipol}})_t = 0$ $\vec{r} = (\cos\theta, -\sin\theta, 0)$

$$\vec{F}_{\text{dipol}} \cdot \vec{r} = \frac{qkP}{R^3} \sin\theta = 0 \Leftrightarrow \begin{cases} \theta = 0 \\ \theta = \pi \end{cases}$$

v) Caràcter de les pos. d'equilibri

Si $g < 0$ i $\theta \approx 0$ amb $\dot{\theta} > 0$, $(\vec{F}_{\text{dipol}})_t < 0 \rightarrow$ restituïtiva

$\Rightarrow \theta = 0$ és eq. estable

En canvi, si $\theta = \pi$, és eq. inestable, $(\vec{F}_{\text{dipol}})_t < 0$

vi) Freqüència de les petites oscil·lacions entorn pos. eq. estable.

Cal trobar l' equació de moviment per θ , que serà de la

forma $\ddot{\theta} = Q(\theta) \approx -\omega^2 \theta$

↳ Taylor

→ Ho troben amb: $MR\ddot{\theta} = m\ddot{q} = (\vec{F}_{\text{neto}})_t = (\vec{F}_{\text{dipol}})_t =$

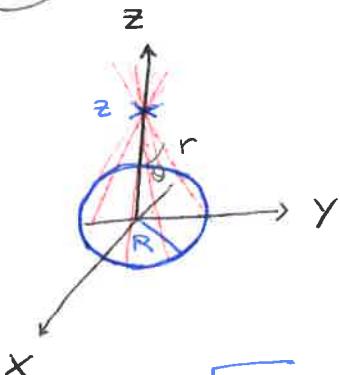
$$= \frac{kqP}{R^3} \sin\theta$$

$$\Rightarrow \ddot{\theta} - \frac{k|q|P}{mR^4} \sin\theta \approx -\frac{k|q|P}{mR^4} \theta \Rightarrow \boxed{\omega = \sqrt{\frac{k|q|P}{mR^4}}}$$

11

a) \vec{E} ? v? a $(0, 0, z)$

cada punt, $dq \Rightarrow d\vec{E} = k \frac{dq}{r^2} \hat{r}$
 ↳ Element de càrrega



$$\Rightarrow \vec{E} = \int d\vec{E} = (0, 0, E)$$

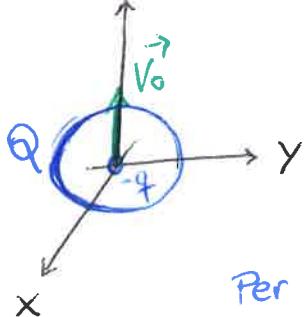
$$\boxed{E = \int dE_z = \int dE \cos \theta = \int dE \frac{z}{r} = \int_{\text{anell}} k \frac{dq}{r^2} \frac{z}{r} =}$$

$$= \frac{kz}{r^3} \int_{\text{anell}} dq = \underbrace{\frac{kQz}{(R^2+z^2)^{3/2}}}_{Q} = \boxed{k \frac{Qz}{(R^2+z^2)^{3/2}}}$$

$$\boxed{V = \int_{\text{anell}} dV = k \frac{1}{r} \int_{\text{anell}} dq = \boxed{\frac{kQ}{(R^2+z^2)^{1/2}}}}$$

$$dV = k \frac{dq}{r}$$

b) $-q, z_0=0, \vec{V}_0 = (0, 0, v_0)$



i) s'allunyarà indefinidament si

$$v_0 > V_e = \sqrt{\frac{2kQq}{mR}}$$

Per conservació de l'energia mecànica:

$$E_T = \frac{1}{2} mv^2 + U(z), \quad U(z) = -q V(z) \quad i \text{ volem } E_m > 0.$$

$$\Rightarrow E_m = \frac{1}{2} mv_0^2 - k \frac{Qq}{(R^2+z^2)^{1/2}} = \frac{1}{2} mv_0^2 - k \frac{Qq}{R} \gg 0 \Rightarrow$$

$$\Rightarrow \boxed{v_0 \geq \sqrt{\frac{2kQq}{mR}}}$$

ii) Si $v_0 < v_e$. z_{\max} ?

$\times \quad V=0, z_{\max}$

$$\frac{1}{2}mv_0^2 - k \frac{Qq}{R} = - \frac{kQq}{(R^2 + z_{\max}^2)^{1/2}} \rightarrow$$

$\rightarrow z_{\max} = R \sqrt{\left(1 - \frac{v_0}{v_e}\right)^{-2} - 1}$

c)

i) V al centre?

$-q, v_0 = 0$

$$U(z_0) = \frac{1}{2}mv^2 + U(0)$$

$$\rightarrow V = \sqrt{2k \frac{Qq}{m} \left(\frac{1}{R} - \frac{1}{(R^2 - z_0^2)^{1/2}} \right)}$$

ii) $z_0 \ll R$, período T del mov. oscilatorio de $-q$.

$$U(z) = -k \frac{Qq}{(R^2 + z^2)^{1/2}}, \quad U(z) \approx U(0) + \frac{dU}{dz} \Big|_{z=0} \cdot z + \frac{1}{2} \frac{\partial^2 U}{\partial z^2} \Big|_{z=0} z^2 + O(z^3)$$

(TAYLOR)

$$\frac{dU}{dz} \Big|_{z=0} = k \frac{qQz}{(R^2 + z^2)^{3/2}} = 0, \quad \frac{d^2U}{dz^2} \Big|_{z=0} = \frac{kQq(R^2 - 2z^2)}{(R^2 + z^2)^{5/2}} = \frac{kQq}{R^3}$$

$"k"$ de MHS

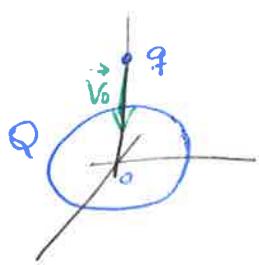
Alejados,

$$U(z) \approx U(0) + \frac{1}{2} k \frac{Qq}{R^3} z^2 \quad \left(U(x) = \frac{1}{2} kx^2 \right)$$

$$T = 2\pi \sqrt{\frac{m}{k}} \rightarrow T = 2\pi \sqrt{\frac{mR^3}{kQq}}$$

d)

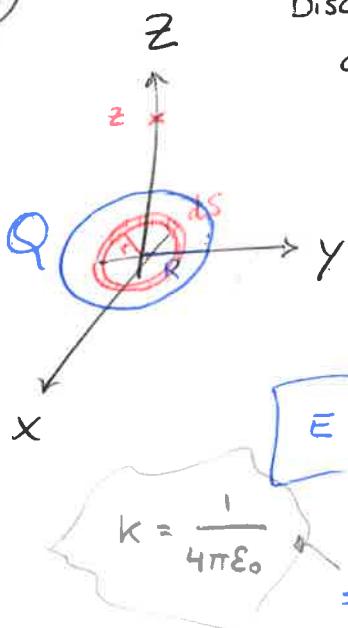
$$U(z) = q V(z) = k \frac{Q z}{(R^2 + z^2)^{1/2}}$$



NO ACABAT

(12)

Disc \rightarrow anells de radi r i gruix dr amb càrrega $dQ = \sigma 2\pi r dr$ ($\sigma = \frac{Q}{\pi R^2}$ dens. sup.)



$$s = \pi r^2 \rightarrow ds = \left(\frac{ds}{dr} \right) dr = 2\pi r dr$$

$$\text{anell } dQ \rightarrow dE = k \frac{dQ z}{(r^2 + z^2)^{3/2}} \quad (\text{Per l'ex 11})$$

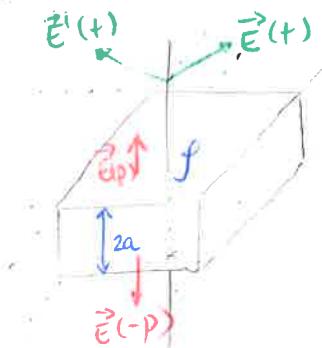
$$E = \int dE = \int_0^R \frac{k z \sigma 2\pi r dr}{(r^2 + z^2)^{3/2}} = \dots = k \sigma 2\pi z \left(\frac{1}{|z|} - \frac{1}{(R^2 + z^2)^{3/2}} \right) =$$

$$= \frac{\sigma}{2\epsilon_0} \left(\frac{z}{|z|} - \frac{z}{\sqrt{R^2 + z^2}} \right)$$

$$V = \int dV = \int_0^R k \frac{dQ}{\sqrt{r^2 + z^2}} = \frac{\sigma}{2\epsilon_0} \left(\sqrt{R^2 + z^2} - |z| \right)$$

14

Com és el camp?



Obs: que la distribució de càrrega és invariant per rotacions entorn eixos ⊥ a la placa (infinita)

Si $\vec{E}'(+)$ és el camp després d'una rotació,

observem que $\vec{E}'(+) = \vec{E}(+)$ per la obs. anterior.

$$\text{Aleshores, } \vec{E}(P) = E(r) \hat{k}$$

Obs: La distribució de càrrega també és invariant per translacions de la placa al pla XY

Per tant, $\vec{E}(P) = \vec{E}(Q)$, on $P = (x, y, z)$; $Q = (x', y', z)$

$$\text{Llavors, } \vec{E}(r) = E(z) \hat{k}$$

Obs: Si li donem la volta a la placa:

$$\text{Observem que } \vec{E}(-p) = -\vec{E}(p)$$

Aleshores, veiem que:

$$\vec{E}(r) = E(z) \hat{k}, \text{ amb } E(z) \text{ senar.}$$

Sabent com és el camp, cal calcular el camp a l'interior de la placa:

Sigui $z' \in (0, a)$:

$$\oint_{S_{z'}} \vec{E} \cdot d\vec{s} = \int_{T_{\text{sup}}} \vec{E}_p d\vec{s} + \int_{T_{\text{inf}}} \vec{E} d\vec{s} =$$

$$E(z) = \frac{\rho z}{\epsilon_0}$$



$$= \int_{T_{\text{sup}}} E(z') dS + \int_{T_{\text{inf}}} E(z'') dS = 2E(z') \cdot A$$

$$\Rightarrow 2E(z') A = \frac{\rho (2z' A)}{\epsilon_0}$$



Anem a veure quin és el camp a l'exterior de la placa:

Observem que $E(z=0) = 0$

Apliquem Gauss a $\tilde{S}_{z'} = \text{sup. del prisma amb Tint a } z=0$

i $T_{\text{sup}} \text{ a } z=z' > a$.

$$\begin{aligned} \text{Inf: } E(z) &= 0 \\ \text{lateral: } \vec{E} &\perp d\vec{S} \Rightarrow \int \vec{E} \cdot d\vec{S} = 0 \end{aligned}$$

Aleshores,

$$\begin{aligned} \oint_S \vec{E} \cdot d\vec{S} &= \int_{T_{\text{sup}}} E(z') dS = E(z') A = \frac{Q_{\text{int}}(\tilde{S}_{z'})}{\epsilon_0} = \\ &= \frac{faA}{\epsilon_0} \Rightarrow E(z') = \frac{fa}{\epsilon_0} \quad (z' > a) \end{aligned}$$

Per tant:

$$\vec{E}(r) = \begin{cases} \frac{fa}{\epsilon_0} \hat{k} & \text{si } z > a \\ \frac{fz}{\epsilon_0} \hat{k} & \text{si } -a < z < a \\ -\frac{fa}{\epsilon_0} \hat{k} & \text{si } z < -a \end{cases}$$

$E \perp \text{pla}$

Ara, calclem el potencial:

Preneu l'origen de potencial a $z=0$ (Els plans horizontals són equipotencials)

Si $z' \in (0, a)$:

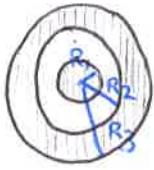
$$V(z') = - \int_{z=0}^{z=z'} \vec{E} \cdot d\vec{r} = - \int_0^{z'} E(z) dz =$$

$$= - \int_0^{z'} \frac{fz}{\epsilon_0} dz = - \frac{fz'^2}{2\epsilon_0}$$

Si $z' \in (0, a)$ (igual si $z' \in (-a, 0)$)

$$\begin{aligned} z' > a: V(z') &= \int_{z=0}^{z=z'} \vec{E}(r) \cdot d\vec{r} - \int_{z=a}^{z=z'} \vec{E} \cdot d\vec{r} = \frac{-fa^2}{2\epsilon_0} - \int_a^{z'} \frac{fa}{\epsilon_0} dz = \\ &= \left[\frac{fa^2}{2\epsilon_0} - \frac{fa}{\epsilon_0} z \right]_a^{z'} \end{aligned}$$

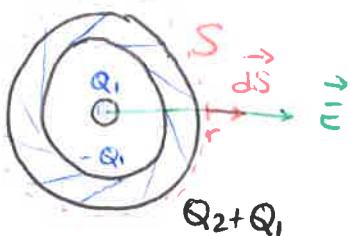
18

 R_1, Q_1 (càrrega total) $(R_2, R_3), Q_2$ (càrrega total)càrrega a cada superfície? $\vec{E}(\vec{r})$? $V(\vec{r})$?

Llei de Gauss:

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{int}}{\epsilon_0}$$

$$\left\{ \begin{array}{l} \text{si } r \in (R_2, R_3) \Rightarrow \vec{E} = 0 \Rightarrow \\ \Rightarrow Q_{int} = 0 \Rightarrow Q_1 + Q = 0 \Rightarrow Q = -Q_1 \end{array} \right.$$



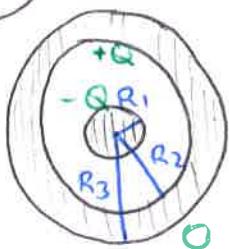
$$\oint \vec{E} \cdot d\vec{S} = \oint E \cdot dS = E \oint dS = ES = E 4\pi r^2 = \frac{Q_{int}}{\epsilon_0} \Rightarrow E = k \frac{Q_{int}}{r^2}$$

$$\Rightarrow E(r) = \begin{cases} 0, & r < R_1 \\ k \frac{Q_1}{r^2}, & r \in [R_1, R_2] \\ 0 & r \in (R_2, R_3) \\ k \frac{Q_1 + Q_2}{r^2} & r \geq R_3 \end{cases}$$

$$V(\infty) = 0 \Rightarrow V(r) = - \int_{\infty}^r E(r') dr'$$

$$V(r) = \begin{cases} r > R_3 \Rightarrow - \int_{\infty}^r k \frac{(Q_1 + Q_2)}{r'^2} dr' = k \frac{(Q_1 + Q_2)}{r} \\ r \in (R_2, R_3) \Rightarrow - \int_{\infty}^r 0 dr' = 0 \\ r \in [R_2, R_3] \Rightarrow - \int_{\infty}^r E(r') dr' = \left(- \int_{\infty}^{R_3} - \int_{R_3}^r \right) E(r') dr' = V(R_3) \\ r \in [R_1, R_2] \Rightarrow - \int_{\infty}^r E(r') dr' = \left(- \int_{\infty}^{R_2} - \int_{R_2}^r \right) E(r') dr' + k \frac{(Q_1 + Q_2)}{R_3} \\ = V(R_2) - \int_{R_2}^r k \frac{Q_1}{r'^2} dr' = \frac{k(Q_1 + Q_2)}{R_3} + k \frac{Q_1}{R} - k \frac{Q_1}{R_2} \\ r \leq R_1 \Rightarrow \left(- \int_{\infty}^{R_1} - \int_{R_1}^r \right) E(r') dr' = V(R_1) = k \frac{Q_1}{R_1} - k \frac{Q_2}{R_2} + k \frac{(Q_1 + Q_2)}{R_3} \end{cases}$$

19



$$\left. \begin{array}{l} Q_1 = -Q \\ Q_2 = Q \end{array} \right\} \Rightarrow \text{Condensador esférico.}$$

Capacitat?

Sabem $C = \frac{Q}{\Delta V} = \frac{Q}{V_+ - V_-}$

$$C = \frac{Q}{V(R_2) - V(R_1)}$$

$$V(R_2) = V(R_3) = k \frac{Q_1 + Q_2}{R_3} \quad (\text{Exercici anterior})$$

$$= k \frac{-Q + Q}{R_3} = 0$$

$$V(R_1) = k \frac{Q_1}{R_1} + k \frac{Q_2}{R_2} + k \frac{Q_1 + Q_2}{R_3} = k \frac{(-Q)}{R_1} - k \frac{(-Q)}{R_2} + 0 =$$

$$= kQ \left(\frac{1}{R_2} - \frac{1}{R_1} \right)$$

$$\Rightarrow V(R_2) - V(R_1) = kQ \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = kQ \cdot \frac{R_2 - R_1}{R_2 R_1}$$

$$\Rightarrow C = \frac{Q}{kQ \left(\frac{R_2 - R_1}{R_2 R_1} \right)} = \underbrace{4\pi\epsilon_0}_{\text{ }} \underbrace{\frac{R_2 R_1}{R_2 - R_1}}_{\text{ }}$$

6. Electrocinetica

(PROBLEMES)

①

$$S = 10^{-6} \text{ m}^2$$

$$I = 1 \text{ A}$$

$$v = v_{\text{mitjana}} e^- ?$$

Cada àtom de Cu aporta $1e^-$

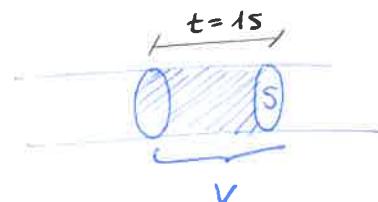
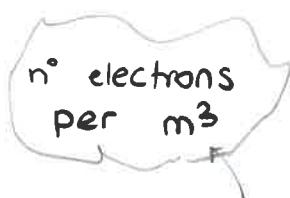
$$\rho_{\text{Cu}} = 8,94 \text{ g/cm}^3$$

$$m_{\text{Cu}} = 63,5 \text{ g/mol}$$

$$N = \#\{e^- \text{ que travessa } S \text{ en } 1s\} = \frac{1}{e} \text{ electrons}$$

$(e = 1,6 \cdot 10^{-19})$

$$= \#\{e^- \text{ del cilindre}\}$$



$$= V_{\text{cilindre}} \cdot N_e = VS N_e$$

Com cada àtom de Cu aporta $1e^-$, $N_e = N_{\text{cu}}$

$$N_{\text{cu}} = \frac{\#\text{àtoms Cu}}{\text{m}^3} = \frac{8,94 \text{ g Cu}}{\text{cm}^3} \cdot \frac{10^6 \text{ cm}^3}{1 \text{ m}^3} \cdot \frac{6,022 \cdot 10^{23} \text{ àtoms}}{63,5 \text{ g Cu}} =$$

$$= 0,848 \cdot 10^{29} \text{ àtoms de Cu/m}^3$$

$$\Rightarrow V = \frac{1}{eSN_e} = \underline{\underline{7,37 \cdot 10^5 \text{ m/s}}}$$

b) Camp elèctric dins del conductor

$$\gamma_{\text{cu}} = 5,96 \cdot 10^7 (\Omega \text{m})^{-1}$$

Si suposem \vec{J} uniforme:

$$\vec{J} = \frac{\vec{I}}{S} = 10^6$$

(En general, $I = \int_S \vec{J} \cdot d\vec{S}$)

Llei d'Ohm: $\vec{J} = \gamma \vec{E}$ la unitat de
 $(\vec{J} = \text{càrrega neta que travessa } S \text{ per unitat de temps})$

$$E = \frac{1}{\gamma} J$$

$$\Rightarrow \boxed{E = \frac{I}{\gamma S} = 0,0168 \text{ V/m}}$$

② En les mateixes condicions que a ①:

Admetem que sobre cada e^- admetem una força del tipus:

$$F = eE - b\dot{x} \quad \left(\begin{array}{l} \text{Aquesta } F \text{ portarà a una vel. límit} \\ \text{de l' } e^- (\nu_\infty) \text{ que serà la } v \text{ de ①} \end{array} \right)$$

Busquem una aproximació de b .

Per la 2a llei de Newton:

$$m\ddot{x} = eE - b\dot{x} \rightarrow \text{EDO d'ordre 2, lineal no homogènia amb coefs. cts.}$$

$$m\ddot{x} + b\dot{x} = eE \quad \textcircled{*}$$

Solució general de la forma:

$$x(t) = x_n(t) + x_p(t)$$

$x_n(t)$: sol. general
($m\ddot{x} + b\dot{x} = 0$)
 $x_p(t)$: sol. particular de $\textcircled{*}$

Com a $x_p(t)$ podem agafar

$$x_p(t) = \frac{eE}{b} t$$

$$\ddot{x}_p(t) = \frac{eE}{b}, \quad \dddot{x}_p(t) = 0$$

$x_n(t)$ és una combinació lineal d'exponentials $e^{\lambda t}$
amb λ les arrels del pol. característic de l'eq. hom.

En aquest cas, el pol. característic és:

$$p(X) = mX^2 + bX = X(mX + b) \quad \begin{cases} \lambda_1 = 0 \\ \lambda_2 = -\frac{b}{m} \end{cases}$$

$$\text{Per tant, } x_n(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} = C_1 + C_2 e^{-\frac{b}{m} t},$$

amb C_1 i C_2 donats per les cond. iniciales.

$$\text{Per tant, } x(t) = C_1 + C_2 e^{-\frac{b}{m} t} + \frac{eE}{b} t$$



→

Les cond. inicials són:

$$\begin{cases} x(0) = 0 \\ \dot{x}(0) = 0 \end{cases} \Rightarrow x(t) = \frac{meE}{b^2} \left(e^{-\frac{b}{m}t} - 1 \right) + \frac{eE}{b} t$$

$$\dot{x}(t) = \frac{Ee}{b} \left(1 - e^{-\frac{b}{m}t} \right) \xrightarrow{t \rightarrow \infty} \frac{eE}{b} = V_\infty$$

Substituint valors:

$$\begin{cases} V_\infty = V_{\text{mitjana}} \text{ de } ① \\ E = \bar{E} \text{ de } ① \end{cases} \Rightarrow b = 0,365 \cdot 10^{-16} \frac{\text{Ns}}{\text{m}}$$

• Temps característic perquè s'estableixi la velocitat límit.

$$\boxed{\dot{x}(t) = \frac{Ee}{b} \left(1 - e^{-\frac{b}{m}t} \right) = V_\infty \left(1 - e^{-\frac{t}{\tau}} \right)}$$

$$\begin{aligned} \tau &= \frac{m}{b} \\ &= 2,5 \cdot 10^{14} \text{ s} \end{aligned}$$

$$\Rightarrow \dot{x}(\tau) = 0,63 V_\infty$$

④

$$P_{\max} (1 \text{ m fil}) = 2 \text{ W}$$

$$I_{\max} = 20 \text{ A}$$

∅ del cable mínim?

Recordem que

$$P = I^2 R$$

$$\Rightarrow P_{\max} = R(1 \text{ m}) \cdot I_{\max}^2 \leq 2$$

L = long
S = secció
γ = conductivitat

$$R = \frac{L}{\gamma S}$$

$$\Rightarrow R(1 \text{ m}) \leq \frac{1}{200} \Rightarrow \frac{L}{\gamma S} \leq \frac{1}{200} \Rightarrow \frac{L}{\gamma \pi \frac{\phi^2}{4}} =$$

$$\begin{aligned} &= \frac{4L}{\gamma \pi \phi^2} \stackrel{L=1}{=} \frac{4}{\gamma \pi \phi^2} \leq \frac{1}{200} \rightarrow \boxed{\phi \geq \sqrt{\frac{800}{\gamma \pi}}} \approx 2 \text{ mm} \end{aligned}$$

6

$$E = 12 \text{ V}$$

$$Q = 160 \text{ Ah}$$

E_{total} de la bateria?

$$E = QE$$

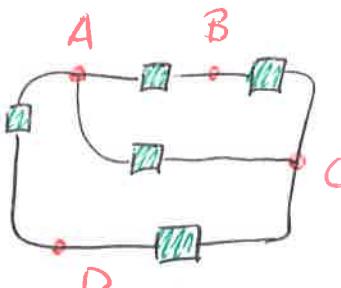
$$E = QE = 6,912 \cdot 10^6 \text{ J}$$

$$\boxed{P = E / \Delta t} \Rightarrow \Delta t = \frac{E}{P} = 46080 \text{ s} \approx 12,8 \text{ h}$$

8

Circuit.

Ex:

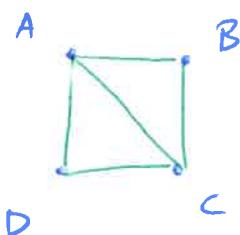


Resoleu el circuit

Volem calcular i i ΔV



A ΔV vèrtex del graf:



1a llei

$$\sum I_a = 0$$

a aresta.

2a llei

A cicle: γ :

$$\sum \Delta V_a = 0$$

a aresta
de γ

$$\begin{cases} I_a > 0 \text{ si entra} \\ I_a < 0 \text{ si surt} \end{cases}$$

\Rightarrow Ens proporciona un sist. sobre determinat.

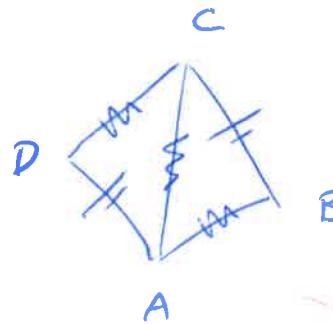
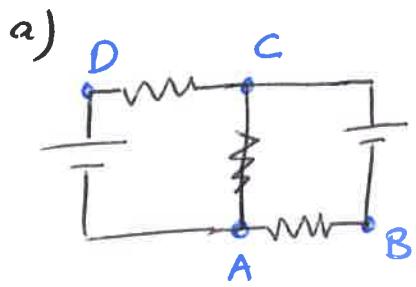
Mètode eficient per resoldre circuits:

Considerem un arbre generador del graf. Per cada aresta que no és de l'arbre, tinc un cicle (fundamental).

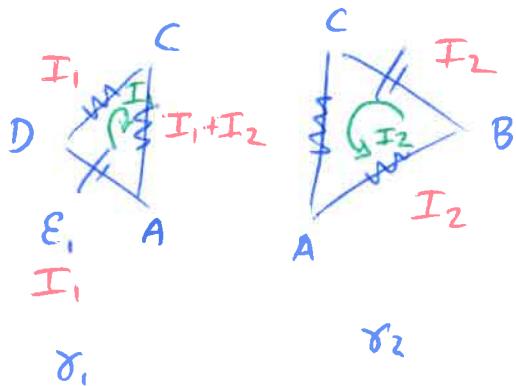
El mètode tracta d'escriure 2a llei de Kirchhoff a cicle fonam.

(Assignant una intensitat i sentit de recorregut a cada cicle)

obs: La I de cada element a l'eq. ha de ser la total.



Tenim 2 cicles fonamentals:

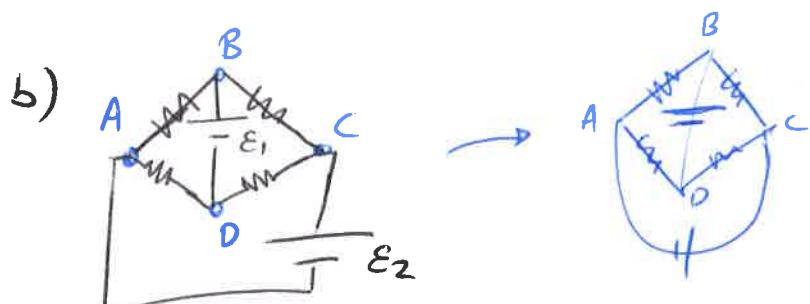


arbre generador
arestes que
creen cicles fon.

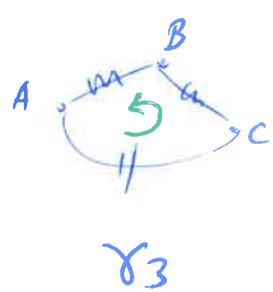
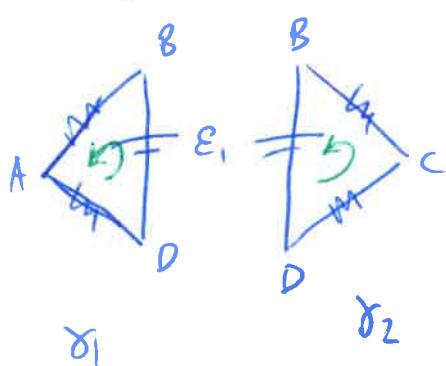
2a llei: $\gamma_1 : E_1 - RI_1 - R(I_1 + I_2) = 0$

$$E_2 - RI_2 - R(I_1 + I_2) = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$\Rightarrow \left. \begin{array}{l} I_1 = \frac{2E_1 - E_2}{3R} \\ I_2 = \frac{2E_2 - E_1}{3R} \end{array} \right. \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

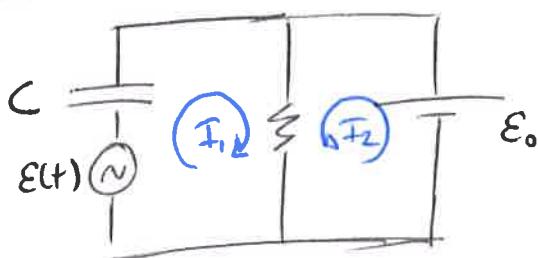


$$\left. \begin{array}{l} E_1 + R(I_1 + I_3) + R(I_1) = 0 \\ -E_1 + R I_2 + R(I_2 + I_3) = 0 \\ E_2 - R(I_3 + I_2) - R(I_3 + I_1) = 0 \end{array} \right\}$$



$$\left. \begin{array}{l} I_1 = \\ I_2 = \\ I_3 = \end{array} \right. \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

10



$$\left\{ \begin{array}{l} E_0 - R(I_1 + I_2) = 0 \\ E(t) - \frac{Q(t)}{C} - R(I_1 + I_2) = 0 \end{array} \right.$$

$$I_1 + I_2 = \frac{E_0}{R}$$

En un condensador: $I_1 = \frac{dQ}{dt} = C \frac{dV}{dt}$

$$E(t) - \frac{Q(t)}{C} - E_0 = 0 \rightarrow Q(t) = C(E(t) - E_0) = C(E_m \cos wt - E_0)$$

$$\Rightarrow I_1 = \frac{dQ}{dt} = -wCE_m \sin wt$$

Trarem la potència subministrada per un generador.

$$P = IE$$

$$P_{gen\ 1} = I_1 E(t) = -wCE_m^2 \sin wt \cos wt$$

$$P_{gen\ 2} = I_2 E_0 = \left(\frac{E_0}{R} + wCE_m \sin wt \right) E_0$$

$$P_{dissipada\ per\ R} = I^2 R = \left(\frac{E_0}{R} \right)^2 R = \frac{E_0^2}{R}$$

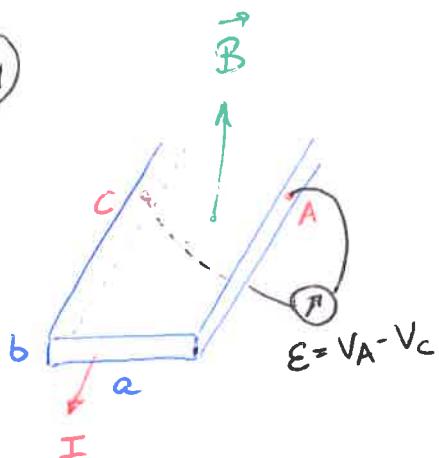
Trarem l'energia emmagatzemada al condensador.

$$E = \frac{Q^2}{2C} = \frac{C^2(E_m \cos wt - E_0)^2}{2C} = C(E_m \cos wt - E_0)$$

7. Magnetostàtica

(PROBLEMES)

④



cinta: $\begin{cases} a = 2 \text{ cm} \\ b = 0,1 \text{ cm} \end{cases}$

corrent $I = 20 \text{ A}$

$B = 2 \text{ T}$

$V_A - V_C = E = 4,17 \mu\text{V} > 0$

- vel. dels portadors ?
- densitat dels portadors ?
- Càrrega + / - ? $|q| = e = 1,6 \cdot 10^{-19} \text{ C}$
- Suposem $q = +e$:

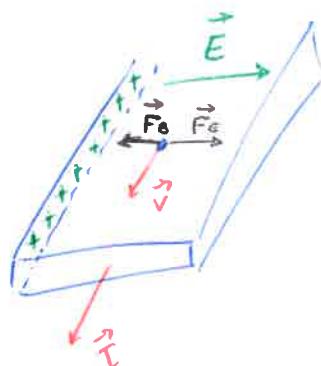
$$\vec{F}_B = q \vec{v} \times \vec{B} \quad , \quad \vec{B} = (0, 0, B) \quad \vec{v} = (v, 0, 0) \quad \leftarrow q \text{ positiva}$$

$$\Rightarrow \vec{v} \times \vec{B} = (0, -vB, 0)$$

$$\Rightarrow \vec{F}_B = q \vec{v} \times \vec{B} = (0, -evB, 0)$$

$$\left. \begin{array}{l} |V_A - V_C| = Ea \\ V_A < V_C \Rightarrow V_A - V_C < 0 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \vec{E} = (0, E, 0)$$

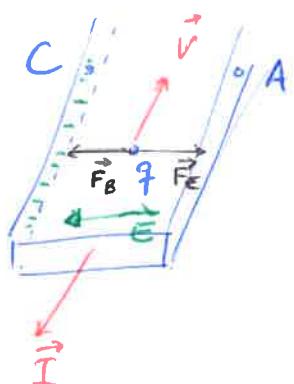


$$\vec{F}_E = q \vec{E} \quad \Rightarrow \text{ambarà un moment que estaran en equilibri}$$

$$\vec{F}_E + \vec{F}_B = 0$$

Initialment E és petit, però va creixent.

- $q = -e$



$$\vec{v} = (-v, 0, 0)$$

$$\vec{v} \times \vec{B} = (0, vB, 0)$$

$$\vec{F}_B = q \vec{v} \times \vec{B} = (0, evB, 0) \quad (q = -e)$$

$$\vec{E} = (0, -E, 0) \implies V_C < V_A$$

$$\implies V_A - V_C > 0 \quad \vec{F}_E = (0, eE, 0)$$

Per l'enunciat, $V_A - V_C > 0$. Llavors, la càrrega ha de tenir signe negatiu.

- A l'equilibri:

$$\vec{F}_B + \vec{F}_E = 0 \implies \vec{F}_B = -\vec{F}_E \implies |\vec{F}_B| = |\vec{F}_E|$$

$$\implies (0, eE, 0) = -(0, -eBv, 0) \implies \boxed{E = BV}$$

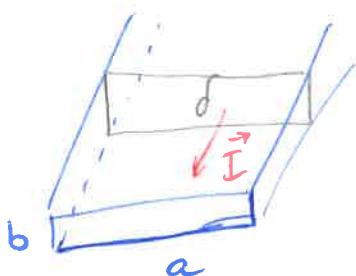
$$V_A - V_C = Ea = Bva = E$$

$$\implies \boxed{v = \frac{E}{Ba} = 1,0425 \cdot 10^{-4} \text{ m/s}}$$

$$\begin{cases} q \cdot n \\ n = n^{\circ} \text{ càrregues} \end{cases}$$

$$\boxed{|J = fV| = qnV}$$

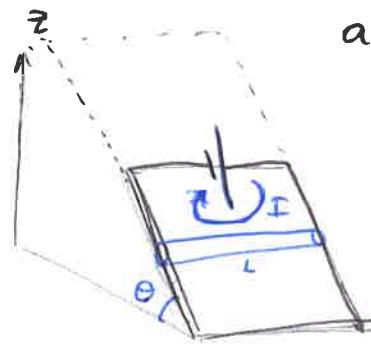
- Trobem la densitat de corrent:



$$\boxed{I = JS}, \quad \boxed{J = env}$$

$$I = envS \implies \boxed{n = \frac{I}{envS} = 5,995 \cdot 10^{28} \text{ e}^-/\text{m}^3}$$

(6)



a)

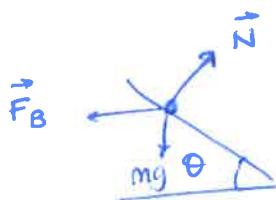
$$\vec{B} = B \vec{k}$$

B per que no llisqui

Coment I en un fil rectilini:

$$\vec{F}_B = I \cdot \vec{l} \times \vec{B}$$

Vista de perfil:



$$\vec{l} \perp \vec{B} \Rightarrow F_B = ILB$$

$$\Rightarrow F_B \cos \theta = mg \sin \theta \Rightarrow$$

$$\Rightarrow ILB \cos \theta = mg \sin \theta \Rightarrow \boxed{B = \frac{mg}{Il} \operatorname{tg} \theta}$$

b) si ara dupliquem \vec{B} ($B = \frac{2mg}{Il} \operatorname{tg} \theta$)Calculeu \vec{a} ?

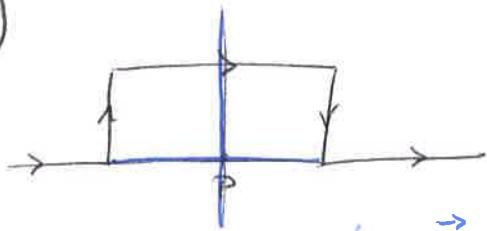
Observem que la varata es mourà cap avant.

$$F_B \cos \theta - mg \sin \theta = m \vec{a} \Rightarrow$$

$$\Rightarrow 2mg \operatorname{tg} \theta \cos \theta - mg \sin \theta = m \vec{a} \Rightarrow g(2 \operatorname{tg} \theta \cos \theta - \sin \theta) = \vec{a}$$

$$\Rightarrow \boxed{\vec{a} = g \sin \theta}$$

(8)

 $\vec{B}(P) ?$

$$\vec{B}(O) = \frac{\mu_0 I}{4\pi} \int_T \frac{\vec{dl} \times |\vec{O} - \vec{r}'|}{|\vec{r}'|^3} = -\frac{\mu_0 I}{4\pi} \int_T \frac{\vec{dl} \times \vec{r}'}{r'^3}$$

- a I_1 , i I_3 : $\vec{dl} \parallel \vec{r} \Rightarrow \vec{dl} \times \vec{r} = 0$

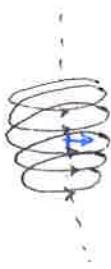
- a I_2 : $\int_{I_2} \frac{\vec{dl} \times \vec{r}}{|\vec{r}'|^3} = \int_0^l \frac{l dy \hat{k}}{(l^2 + y^2)^{3/2}} = \frac{\vec{k}}{l\sqrt{2}}$

- a I_3 = $= \frac{2\vec{k}}{l\sqrt{2}}$

- a I_4 = $= \frac{\vec{k}}{l\sqrt{2}}$

$$\Rightarrow \int_T = \sum_i \int_{I_i} \Rightarrow \vec{B}(O) = -\frac{\mu_0 F}{4\pi} \frac{4\vec{k}}{l\sqrt{2}} = -\frac{\mu_0 I}{2\pi l} \boxed{\vec{k}}$$

(10)



solenoide infinit

n espires / m

Solenoide: Distribució superf. de corrent de densitat:

$$\vec{K} = k \hat{e}_y$$

on k = càrrega neta que atravesà el segment unitat perpendicular al moviment per unitat de temps.

$$(k = nI)$$

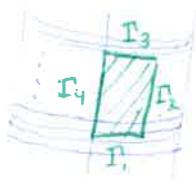




$$\text{Per tant: } \vec{B}(r, \varphi) = B_z(r) \hat{e}_z + B_\varphi(r) \hat{e}_\varphi = B_z(r) \hat{e}_z$$

* Càlcul de $B_z(r)$:

i) Punts a l'interior ($r < R$): Apliquem Ampère a:



$$\circ \text{ A } I_1, I_3: \vec{B} \perp d\vec{l} \Rightarrow \vec{B} \cdot d\vec{l} = 0$$

$$\circ \text{ A } I_2: d\vec{l} = dz \hat{e}_z \quad \vec{B} = B_z(r) \hat{e}_z$$

$$\vec{B} \cdot d\vec{l} = B_z(r) dz$$

$$\Rightarrow \int_{I_2} \vec{B} \cdot d\vec{l} = \int_{z_0}^{z_0+r} B_z(r) dz = B_z(r) r$$

$$\circ \text{ A } I_4: d\vec{l} = dz \hat{e}_z \quad \vec{B} = B_z(0) \hat{e}_z \quad \left. \begin{array}{l} \vec{B} \cdot d\vec{l} = B_z(0) dz \end{array} \right\} \Rightarrow$$

$$\Rightarrow \int_{I_4} \vec{B} \cdot d\vec{l} = \int_{z_0+r}^{z_0} B_z(0) dz = -B_z(0) r.$$

$$\int_I \vec{B} \cdot d\vec{l} = \mu_0 \overset{0}{\underset{S(I)}{\parallel}} I_S = 0 \quad \Rightarrow \left[\int_{I_2} + \int_{I_4} \right] (B_z(r) - B_z(0)) r = 0$$

$$\Rightarrow B_z(r) = B_z(0)$$

Busquem $B_z(0)$ amb Biot-Savart

per calcular el camp a $(0, 0, 0)$:

$$\vec{B}_z(0) = \frac{\mu_0}{4\pi} \int_S \frac{(nI \hat{e}_\varphi) \times (-\vec{r}')}{|\vec{r}'|^3} dS, \text{ on } \begin{cases} \hat{e}_\varphi = (-\sin\varphi, \cos\varphi, 0) \\ \vec{r}' = (R\cos\varphi, R\sin\varphi, z') \end{cases}$$

$$\boxed{B_z(0,0,0)} = \frac{nI\mu_0}{4\pi} \int_{S_L} \frac{R}{(R^2 + z^2)^{3/2}} dS \stackrel{dS = Rd\varphi dz'}{=} \frac{nI\mu_0 R^2}{4\pi} \int_0^{2\pi} d\varphi \int_{-\frac{L}{2}}^{\frac{L}{2}} dz' \frac{1}{(R^2 + z^2)^{3/2}}$$

$$= \dots = \frac{n\mu_0 I}{2} \frac{2 L}{\sqrt{R^2 + (\frac{L}{2})^2}} \xrightarrow{L \rightarrow \infty} \boxed{\mu_0 n I}$$

Tenim que $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_S \frac{\vec{k} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dS$

$(I d\vec{l} \leftrightarrow \vec{k} dS)$ (camp en una distribució superficial de càrrega)

Llei d'Ampère: $\oint_I \vec{B} \cdot d\vec{l} = \mu_0 I_{S(I)}$

on $I_{S(I)} =$ intensitat que atravessa $S(I)$, $I_{S(I)} = \int_S \vec{j} \cdot d\vec{S}$

Forma de \vec{B} deduïda de les sist. de la distr. de corrent:

En principi: $\vec{B}(r, \varphi, z) = B_r(x, \varphi, z) \hat{e}_r + B_\varphi(x, \varphi, z) \hat{e}_\varphi + B_z(x, \varphi, z) \hat{e}_z$

1) La distribució queda invariant per translació en direcció \hat{e}_z
 \Rightarrow Les comp. de \vec{B} no depenen de z .

2) La distribució queda invariant per rotació en Oz
 \Rightarrow Les comp. de \vec{B} no depenen de φ .

3) La distribució queda invertida per rotació d'angle π en eixos horitzontals $\hookrightarrow (\vec{k} = -\vec{k}, \vec{B} = -\vec{B})$
A més, al rotar el camp no varia el vector \hat{e}_r

Aleshores, $B_r = -B_r \Rightarrow B_r = 0$

Per tant, $\vec{B}(r, \varphi) = B_z(r) \hat{e}_z + B_\varphi(r) \hat{e}_\varphi$

* Calcul de B_φ : Apliquem Ampère a $I =$ circumferència en el pla xy centrada a l'origen i de radi $r \neq R$:

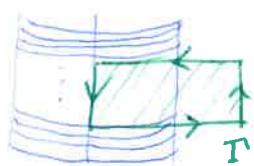
$\oint_I \vec{B} \cdot d\vec{l} = \mu_0 I_{S(I)} = 0$

$\Rightarrow \oint_I \vec{B} \cdot d\vec{l} = \int_0^{2\pi} B_\varphi(r) r d\varphi = 2\pi r B_\varphi(r) = 0 \Rightarrow B_\varphi(r) = 0$

$\left. \begin{cases} \vec{B} \cdot d\vec{l} = B_\varphi(r) r d\varphi \\ r d\varphi \hat{e}_\varphi = d\vec{l} = d\varphi \hat{e}_\varphi \end{cases} \right\}$

←

ii) Punts a l' exterior: Apliquem Ampère:



$$\oint_T \vec{B} \cdot d\vec{l} = \mu_0 I_{S(T)} = -\mu_0 nI$$

• A I₁: $\int_{I_1} \vec{B} \cdot d\vec{l} = \dots = B_z(r)$

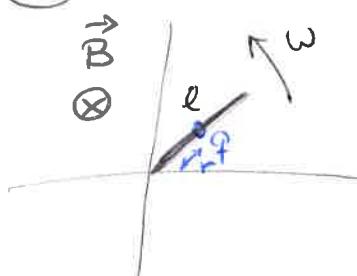
• A I₂: $\int_{I_2} \vec{B} \cdot d\vec{l} = \dots = -\mu_0 nI$

$$\oint_T \vec{B} \cdot d\vec{l} = B_z(r) - \mu_0 nI = -\mu_0 nI \Rightarrow \boxed{B_z(r) = 0}$$

8. Equacions de Maxwell

(PROBLEMES)

2



a) \vec{F} que \vec{B} fa sobre q a distància r

$$\vec{F}_m = q \vec{v} \times \vec{B}$$

$$\vec{B} = (0, 0, -B)$$

$$\vec{v} = rw \hat{e}_\phi$$

$$\rightarrow \vec{F}_m = -qrwB \hat{e}_r \quad (\text{cap al centre si } q > 0)$$

b) ΔV entre O i A en l' equilibri

$$\vec{F}_E = -\vec{F}_B \implies q \vec{E} = -q \vec{v} \times \vec{B}$$

Preneint mòduls:

$$E = rwB \quad (\text{depèn de } r)$$

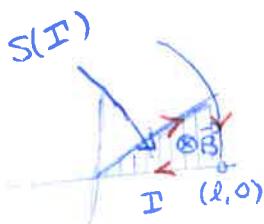
$$d\vec{l} = dr \hat{e}_r$$

$$\vec{E} = rwB \hat{e}_r$$

$$V_A - V_0 = - \int_0^A \vec{E} \cdot d\vec{l} = - \int_0^l rwB dr = -\frac{1}{2} wBl^2$$

c) fem induïda a la barra?

Suposem que tenim un circuit que inclou la barra:



$$E_I = - \frac{d\Phi_I}{dt}, \quad \text{on} \quad \Phi_I = \int_{S(I)} \vec{B} \cdot d\vec{S}$$

orientem I en sentit horari

Amb la orientació induïda a $S(I)$:

$$\left\{ \begin{array}{l} d\vec{S} = -dS \hat{e}_z \\ \vec{B} = -B \hat{e}_z \end{array} \right.$$

$$\Rightarrow \Phi_I = + \int_{S(I)} \vec{B} \cdot d\vec{S} = \int_{S(I)} B dS = BS(\varphi) = \frac{1}{2} Bl^2 \varphi (+)$$

$$\hookrightarrow 2\pi \rightarrow \pi l^2$$

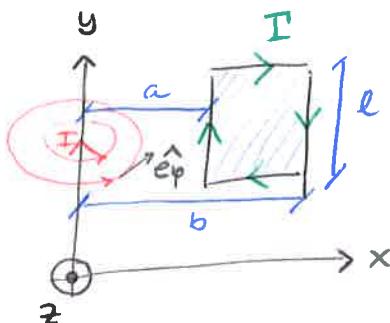
$$\varphi \rightarrow S(\varphi) \Rightarrow S(\varphi) = \frac{1}{2} l^2 \varphi$$

$$\Rightarrow \Phi_I = \frac{1}{2} Bl^2 \varphi(+) = \frac{1}{2} Bl^2 \omega t$$

considerem $\varphi(0) = 0$

$$\Rightarrow \mathcal{E} = - \frac{d\Phi_I}{dt} = \boxed{-\frac{1}{2} Bl^2 \omega l^2 < \mathcal{E}}$$

(4)



camp creat pel fil infinit de corrent

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{e}_z$$

Flux de \vec{B} a través de I:

orientem I en sentit horari i $S(I)$:

$$\vec{dS} = -dS \hat{e}_z$$

$$\vec{B} = -B \hat{e}_z$$

$$\Rightarrow \boxed{\Phi_I = \int_{S(I)} \vec{B} \cdot d\vec{S} = \iint_{S(I)} \frac{\mu_0 I}{2\pi r} dx dy = \int_a^b dx \int_0^l dy \frac{\mu_0 I}{2\pi x} =}$$

$\hookrightarrow \vec{B} \cdot d\vec{S} = \frac{\mu_0 I}{2\pi x} dS$
" " dydx

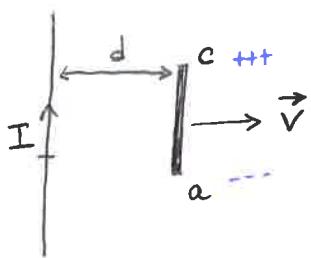
$$= \boxed{\frac{\mu_0 l I}{2\pi} \ln\left(\frac{b}{a}\right)}$$

Suposem $I = I_0 \sin \omega t$ i busquem la fem induida

a I:

$$\boxed{\mathcal{E} = - \frac{d\Phi_I}{dt} = - \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right) I_0 \omega \cos \omega t}$$

(6)

 \mathcal{E} induida.

$$\vec{B} ? \quad \vec{F} ? \quad \vec{E} ? \quad (F_B = F_E) \quad \mathcal{E} ? \quad (= \Delta V)$$

$$\vec{B}(a) = \frac{\mu_0 I}{2\pi d}$$

$$(B \cdot L = B \cdot 2\pi d = \mu_0 I)$$

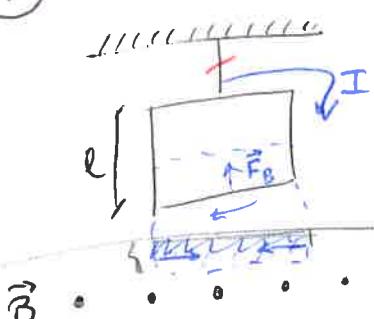
$$\text{Sabem } F_m = q \vec{v} \times \vec{B} = \bar{e} v B \vec{j} \quad ; \quad \vec{E} = -\vec{E}_j$$

$$\text{Atribuirem un moment que } \vec{F}_{el} = -\vec{F}_m \Rightarrow \vec{E} = v B \vec{i} \Rightarrow$$

$$\Rightarrow \mathcal{E} = v B = \frac{\mu_0 V I}{2\pi d}$$

$$\mathcal{E} = \Delta V = \int_C^A \frac{\mu_0 I}{2\pi d} v \, dl \Rightarrow \left(\mathcal{E} = \frac{\mu_0 I}{2\pi d} v l \right) = V_A - V_C$$

(7)



$$\vec{F}_B = I \vec{l} \times \vec{B} \Rightarrow F_B = ILB$$

$$mg - ILB = m \frac{dv}{dt}$$

$$I = \frac{\mathcal{E}}{R} \Rightarrow \mathcal{E} = -\frac{d\phi}{dt}$$

$$\text{on } d\phi = B \cdot ds \\ = B \cdot l v dt$$

$$\Rightarrow \mathcal{E} = -\frac{d\phi}{dt} = -\frac{Blv dt}{dt} = -Blv$$

$$\Rightarrow I = \frac{-Blv}{R} \Rightarrow mg + \frac{Blv}{R} = m \frac{dv}{dt} \Rightarrow$$

$$\Rightarrow \frac{dv}{dt} + \frac{B^2 l^2}{m R} v = g \Rightarrow v(t) = K e^{-\frac{B^2 l^2}{m R} t} + v^{(p)}$$

on $v(p) = \frac{mgR}{B^2 l^2}$

$\Rightarrow v(t) = K e^{-\frac{B^2 l^2}{m R} t} + \frac{mgR}{B^2 l^2} \left(1 - e^{-\frac{B^2 l^2}{m R} t}\right)$

